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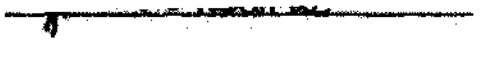

SIMULTANEOUS HEAT, MOMENTUM AND MASS TRANSFER IN THE THROUGH-FLOW DRYING OF AGRICULTURAL PRODUCTS

A THESIS

Presented to

The Faculty of the Graduate Division

by

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SIMULTANEOUS HEAT, MOMENTUM AND MASS TRANSFER IN THE THROUGH-FLOW DRYING OF AGRICULTURAL PRODUCTS

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LIST OF SYMBOLS

a	the interfacial area per unit volume of bed, ft^2/ft^3
A	constant in Eq. (32), page 77
A_o	cross sectional area of bed, ft^2
B	constant in Eq. (32), page 77
c	molar concentration of mixture, moles of mixture/ft^3
c_A	apparent molar concentration of water in wet material, lb-mole of water/ft^3
c_h	apparent molar concentration of dry material, lb-mole/ft^3
c_i	apparent molar concentration of species i, lb-mole of i/ft^3
c_j	apparent molar concentration of species j, lb-mole of j/ft^3
c_{ph}	specific heat of dry material, BTU/lbm °R
c_{pw}	specific heat of liquid water, BTU/lbm °R
$\tilde{C}_{ph}$	constant pressure molar specific heat of dry material, BTU/lb-mole °R
$\tilde{C}_{pAl}$	constant pressure molar specific heat of liquid water, BTU/lb-mole °R
$\tilde{C}_{pA}$	constant pressure molar specific heat of water vapor, BTU/lb-mole of water °R
$\tilde{C}_{pB}$	constant pressure molar specific heat of air, BTU/lb-mole of air °R
$\tilde{C}_{vAl}$	constant volume molar specific heat of liquid water, BTU/lb-mole °R
$\tilde{C}_{vh}$	constant volume molar specific heat of dry hay, BTU/lb-mole °R
C	constant in Eq. (32), page 77
C_1	group of terms in Eq. (46), page 83

LIST OF SYMBOLS (Continued)

C_2	group of terms in Eq. (46), page 83
C_3	group of terms in Eq. (46), page 83
C_4	group of terms in Eq. (46), page 83
C_5	group of terms in Eq. (46), page 83
C_6	group of terms in Eq. (46), page 83
D	constant in Eq. (32), page 77
$\mathcal{D}_{AB}$	diffusivity of air-water vapor mixture, ft^2/minute
D_p	equivalent particle diameter, defined by Eq. (29), page 75
E	constant in Eq. (32), page 77
f	friction factor defined by Eq. (27), page 74
F	constant in Eq. (32), page 77
g_c	constant, 32.2 lbm-ft/lbf-sec^2
G	mass flow rate per square foot of bed, lbm/min ft^2
Gl	constant in Eq. (32), page 77
Gr	constant, see Eq. (31), page 76
$h_{x,loc}$	convective heat transfer coefficient, BTU/ft^2minute ^oR
H	constant in Eq. (32), page 77
$\bar{H}_i$	partial molar enthalpy of species i in the air-water vapor mixture, BTU/lb-mole of i
$\bar{H}_{AS}$	partial molar enthalpy of species i at the drying surface of the wet material, BTU/lb-mole of i
H_{fg}	molar latent heat of vaporization of water vapor at the temperature of drying surface, BTU/lb-mole
K	constant in Eq. (32), page 77
$k_{x,loc}$	local convective mass transfer coefficient, lb-mole/mixture ft^2
ℓ	length of particle, ft

L	depth of bed, ft
L1	constant in Eq. (32), page 77
m_h	mass of dry material, lbm
m_w	mass of moisture contained in bed, lbm
M	moisture content, lbm moisture/lbm dry material
M_A	molecular weight of water, lbm/lb-mole
M_B	molecular weight of air, lbm/lb-mole
M_c	critical moisture content, lbm moisture/lbm dry material
M_e	equilibrium moisture content, lbm moisture/lbm dry material
M_i	initial moisture content, lbm moisture/lbm dry material
M1	constant in Eq. (32), page 77
n	number of particles
n_o	direction cosine of unit normal
N_i	molar flux of species i, lb-mole of i/minute ft^2
N_A	molar flux of water vapor, lb-mole of moisture/minute ft^2
N_B	molar flux of air, lb-mole of air/minute ft^2
P	total pressure of mixture, lbf/ft^2
P_{AS}	vapor pressure at the drying surface, atms
Pr	Prandtl number
P_o	porosity of bed, volume of voids/volume of bed
P_1	initial pressure in tank 1 in Figure 5, in H_2O
P_3	final pressure in both tanks, in H_2O
ΔP	pressure drop across the bed of material, lbf/ft^2
ΔP_d	pressure drop across the dry bed of material, lbf/ft^2
ΔP_i	initial pressure across the bed of material, lbf/ft^2
Q	volumetric flow rate of air, ft^3/minute

LIST OF SYMBOLS (Continued)

r	(M - Me)/(Mc - Me)
R	universal gas constant, 1545 lbf ft/lb mole °R
Re	modified Reynolds number
R_i	molar rate of mass generation per unit volume, lb-mole of i/ft^3 minute
R_j	molar rate of mass generation per unit volume, lb-mole of j/ft^3 minute
Sc	Schmidt number
T	temperature of mixture, °R
T_s	temperature of the drying surface, °R
T_o	initial temperature in the bed, °R
T_1	temperature of the mixture at entrance to the bed, °R
U_{al}	molar internal energy of the liquid water, BTU/lb-mole
U_h	molar internal energy of dry material, BTU/lb-mole
$\bar{U}_i$	partial molar internal energy of species i, BTU/lb-mole
$\bar{U}_j$	partial molar internal energy of species j, BTU/lb-mole
V	total volume of bed, ft^3
V_m	volume occupied by the material, ft^3
V_v	volume of voids in the bed, ft^3
V_1	volume of tank 1 in Figure 5
V_2	volume not occupied by test material in tank 2 in Figure 5
W_b	molar flux of air, lb-mole of air/minute
x_A	mole fraction of the water vapor in the mixture, lb-mole moisture/lb-mole mixture
x_{AO}	initial mole fraction of the water vapor in the mixture, lb-mole moisture/lb-mole mixture

LIST OF SYMBOLS (Continued)

x_{A1}	mole fraction of the water vapor at the entrance to the bed, lb-mole moisture/lb-mole mixture
x_{AS}	mole fraction of the water vapor at the surface of the wet material, lb-mole moisture/lb-mole mixture
Y_1	group of terms in Eq. (32), page 77
Y_2	group of terms in Eq. (32), page 77
Y_3	group of terms in Eq. (32), page 77
Y_4	group of terms in Eq. (32), page 77
Z	distance measured from entrance of bed in direction of air flow, ft
$(\frac{\partial M}{\partial \tau})_c$	rate of drying at the critical moisture content, lbm moisture/lbm dry material minute
α	group of terms in Eq. (32), page 77
β	slope of straight line approximating the falling rate period as defined by Eq. (51), page 88
ρ	density of air, lbm/ft^3
ρ_h	density of hay, lbm/ft^3
ρ_w	density of liquid water, lbm/ft^3
τ	time, minutes
μ	viscosity of air, lbm/ft sec

SUMMARY

Simultaneous heat, momentum and mass transfer in the through-flow drying of beds of hay was investigated analytically and experimentally. Analytical equations were obtained in general form and solved for the initial adjustment and constant rate period by numerical methods on a digital computer. The equations were then simplified by neglecting terms which were found by the solution of the general equations to be insignificant. The simplified equations were integrated to give an expression for the drying time between the initial and critical moisture content. An approximate solution for the falling rate period of drying was included for completeness. The two expressions were combined to obtain a closed form expression for the total drying time.

The experimental study consisted of investigating the effect on the drying rate of air temperature and flow rate, initial moisture content of material, bed thickness, bed density, and length of material. The ranges over which the variables were tested were:

(i) Temperature, Room - 335°F

(ii) Flow Rate, 300 - 500 cfm

(iii) Initial Moisture Content, 119 - 447 per cent

(iv) Bed Thickness, 1 - 4 inches

(v) Bed Density, (wet) 9.16 - 13.75 lbm/ft^3

(vi) Particle Length, 0.5 - 16 inches

It was found that these variables could be successfully correlated by use of a modified Reynolds number and drag coefficient. The pressure

drop at constant flow rate was found to vary approximately linearly with moisture content between the initial and critical moisture content.

The total drying time was found to increase with increases in (i) bed thickness; (ii) bed density; (iii) initial moisture content; and, (iv) length of material; and to decrease with increases in (i) flow rate; (ii) temperature; and (iii) concentration of water vapor in the incoming stream.

The drying time between the initial and critical moisture content, as predicted by the analytical equations, agreed to within 35 per cent of the experimental drying time. The analytical drying times for 70 per cent of the runs were within 10 per cent of the experimental values.

CHAPTER I

INTRODUCTION

There presently exists a great need for a process which would completely mechanize the harvesting, storage, and feeding facilities of hay. Similar agricultural products could also be benefited by the development of such a process.

Problem

In the traditional method of harvesting, hay is cut and allowed to lie in the field and cure for a period of from one to three days. During this period the action of wind and sun reduces the moisture content* (dry basis) to approximately fifteen per cent. The initial moisture content may be as high as 500 per cent in some climates. The hay is then raked and either stood in shocks or baled (usually baled). These bales are then placed in a storage facility until time for feeding.

This traditional method of harvesting has three disadvantages which may soon prohibit its usage. The conditions which normally permit natural drying in the field coincides with a stage of growth of the hay which is so advanced that the material is initially of low food value, especially in protein content. Also, due to the action of the wind, much of the high protein of the leaf portion of the plant is lost. When hay is field dried, there is a marked bleaching and loss of green color.

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*See Chapter II, page 5 for definitions.

This loss of color increases with the length of exposure and strength of sunlight. Associated with this color loss is a loss in carotene (Vitamin A), a substance of vital importance in preventing certain livestock diseases in cooler climates. Also, the traditional method is very laborious and thus expensive. And thirdly, each year over one fourth of all of the material grown in the United States is lost due to improper field drying.

The inherent disadvantages of the harvesting methods of today suggest a seemingly simple solution -- a process in which the material is cut, artificially dried and then formed into a pellet* or wafer** so that it can be mechanically stored and fed. Various research groups, corporations, and individuals have been working on such a process for the past decade. Many independent designs have been developed but they all have one common disadvantage which prohibits their widespread application -- cost of operation. The complete study of this type of process is beyond the scope of this investigation; therefore, only one portion will be investigated here, that of artificial drying.

Artificial Drying

In the drying of hay the first change that occurs is a reduction of the moisture to such a level that the cells are killed. The material

---

*Hay pellets are normally defined as an agglomerated ground forage in which the particle size is generally less than 3/8 inch in length. The superscript numbers refer to the references listed in the Bibliography, Appendix F, page 139.

**Hay wafers are normally defined as an agglomerated unground forage which has some fibers equal to or greater than the length of its minimum cross section dimensions.

can then be stored safely without any other changes taking place, such as those due to the action of molds.[2] When hay is first cut the plants of which it consists are still alive, and the vital processes will continue until the cells die. The most important of these processes is respiration, which takes place in the presence of air and results in an oxidation of the nitrogen-free extractives or soluble carbohydrates.[2]

In natural field drying the ready access to air allows respiration to proceed. If the hay is dried very rapidly in intense sunlight the loss in food value may be low, but usually the process extends over a period of some days and the loss of food value as well as material in the form of small leaves and stems is appreciable. By cutting the material and transporting it directly to a drier where it can be dried very rapidly, these losses can be reduced to a negligible figure.

It has been shown that the amounts of crude protein and starch are almost doubled when hay is properly artificially dried.[2] The carotene content can be completely preserved, while in natural drying it is almost always completely lost. With artificial drying, the loss of material (leaves and stems) is negligible and fermentation during storage is completely prevented. Under present methods of drying this loss amounts to 28 per cent of the hay crop each year.

Another important advantage of artificial drying is that it leaves the material in a form ready for processing into pellets or wafers, or storage in bags or bales.

In summary, it can be said that artificial drying, if properly carried out, will give a product which supplies as much energy and digestible protein as would the original fresh crop but in a more condensed form

and in a state ready for mechanical processing and storing.

Purpose of this Investigation

It is the purpose of this investigation to analytically and experimentally study the simultaneous momentum, heat and mass transfer in the through-flow drying of a bed of hay under various conditions of air temperature, velocity, initial moisture content, bed thickness, length of material and density of packing. The analytical results will consist of the prediction of drying rates during the initial adjustment and constant rate periods of drying and a correlation of variables relating the pressure drop through the bed. The results of these studies will then be used to aid in the interpretation of the experimental results obtained under different drying conditions.

Application

In 1963, over one billion acres of hay were grown in the United States. It is hoped that the results of this investigation may be used in the development of a commercial agricultural drier which can efficiently dry hay at a cost per ton sufficiently low to enhance its widespread usage. Such a drier, along with the related pelleting equipment, would give the farmer a manufactured product (pellets) which he could either use himself or sell commercially as wheat and grain are currently sold.

CHAPTER II

BACKGROUND

Mechanism of Drying

Drying refers to the removal of liquid from a solid by thermal means. This definition distinguishes drying from mechanical methods of removing liquids from solids, but it does not differentiate drying from evaporation in which heat is used to evaporate large amounts of water from solution or slurries.[3] Drying is usually differentiated from evaporation by the equipment used. Evaporation processes generally remove much larger quantities of liquid than do drying processes.

The following definitions, restricted to air water-vapor mixtures, as given by Perry,[3] are generally accepted in drying:

> Bound moisture is that liquid held by a solid which exerts a vapor pressure less than that of the pure liquid at the same temperature. Liquid may become bound by retention in small capillaries, solution in cell or fiber walls, homogeneous solution throughout the solid, and by chemical or physical adsorption on solid surfaces. Bound moisture can be removed from a solid only under specific conditions of humidity in the external surroundings.
>
> Capillary flow is the flow of liquid through the interstices and over the surface of a solid, caused by liquid-solid molecular attraction.
>
> Concentration denotes the amount of water vapor actually present in a mixture.
>
> Constant-rate period is that drying period during which the rate of water removal per unit of drying surface is constant.
>
> Dry-weight basis indicates the moisture content of wet solid as pounds of water per pound of dry solid.

Equilibrium moisture content is the moisture content to which a given material can be dried under specific conditions of air temperature and concentration.

Falling-rate period is that drying period during which the instantaneous drying rate continually decreases.

Free moisture content is that liquid which is removable at a given temperature and concentration. It may include bound and unbound moisture.

Hygroscopic material is one that may contain bound moisture.

Initial moisture distribution refers to the moisture distribution throughout a bed at the start of drying.

Moisture content of a solid is usually expressed as moisture weight per unit weight of dry or wet solid.

Non-hygroscopic material is one that can contain no bound moisture.

Unbound moisture in a hygroscopic material is that moisture in excess of the equilibrium moisture content which experts a vapor pressure equal to that of pure liquid at the same temperature. All water in a non-hygroscopic material is unbound water.

Wet-weight basis expresses the moisture in a material as a percentage of the weight of the wet solid.

The factors governing the rate at which a substance dries are numerous; however, the drying periods can be grouped into four distinct categories. These will be discussed in the following paragraphs.

To assist in the explanation of the regions of drying, a typical equilibrium moisture curve is shown in Figure 1. If a substance is exposed to a stream of air having a relative humidity A, it will loose or gain moisture until its equilibrium moisture content corresponding to point D on the curve is eventually reached. Further exposure to the air will not affect the moisture content of the substance. It is seen from this that the final state of dryness depends on the prevailing conditions of the air stream and can only be reduced by reduction in the relative humidity

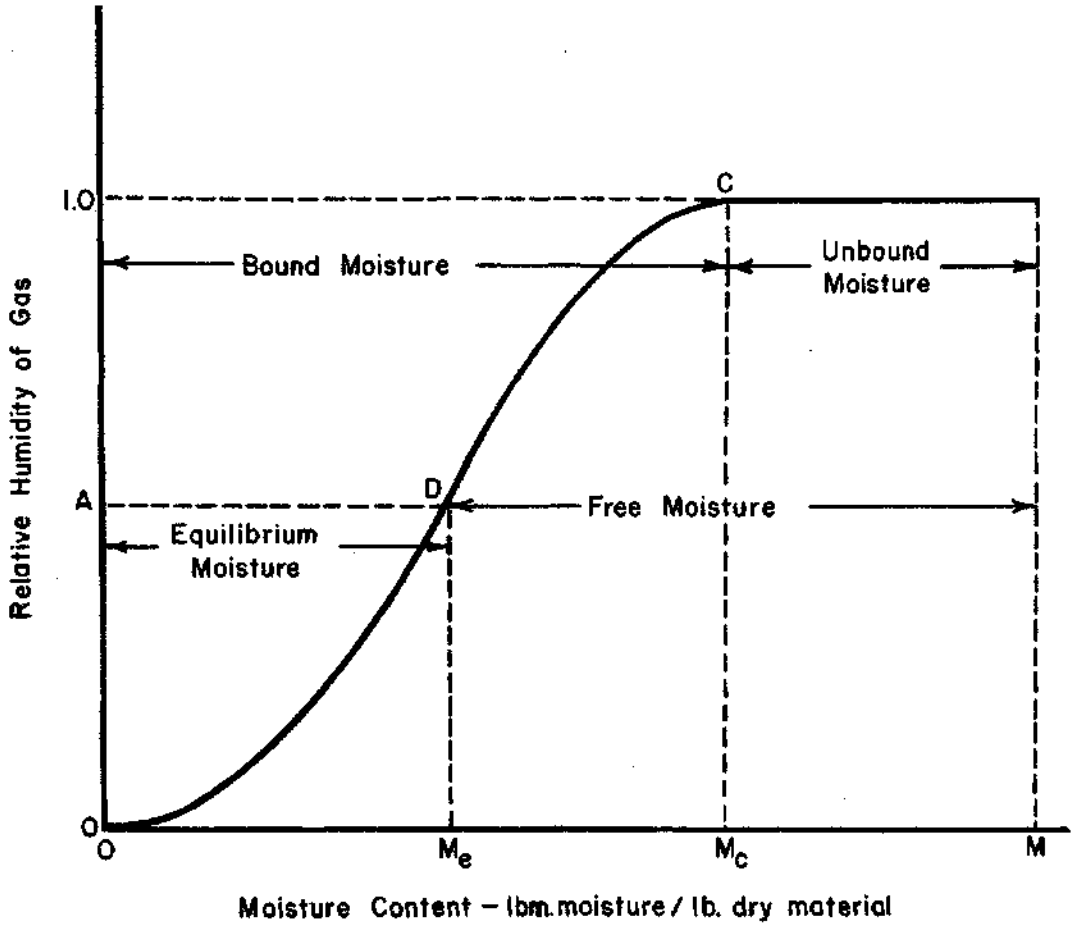


Figure 1. Equilibrium Moisture Curve.

of the air. The moisture contained in a substance up to a concentration corresponding to point C on the curve is referred to as bound moisture.

The drying rates associated with the regions of drying for a thin bed are shown schematically in Figure 2. In this investigation, these rates will be referred to as the local rate of drying. Upon exposure to an air stream of constant temperature and concentration, the surface temperature of a substance with surface moisture will begin to adjust to a local equilibrium wet bulb temperature. During this adjustment, if the surface temperature is initially lower than the corresponding wet bulb temperature, the drying rate will increase as drying proceeds as shown in portion AB on the curve. If the surface temperature is higher than the corresponding wet bulb temperature, the drying rate will decrease as drying proceeds, as shown in period A'B. This period is referred to as the initial adjustment period.

The portion BC on the curve represents the local constant rate period of drying. During this period a balance between heat transfer to and mass transfer from the surface occurs and a constant temperature and concentration at the surface results. This leads to the constant drying rate which depends on the heat and mass transfer coefficients and the corresponding gradients associated with these; i.e., the rate depends on the rate of diffusion of water vapor through the surface air film out into the main body of air, and thus, it depends on the concentration, temperature and velocity of the air.

Point C on the curve in Figure 2 refers to the local critical moisture content. The constant rate period ends when the moisture content at the surface reaches some specific value. If the rate of drying

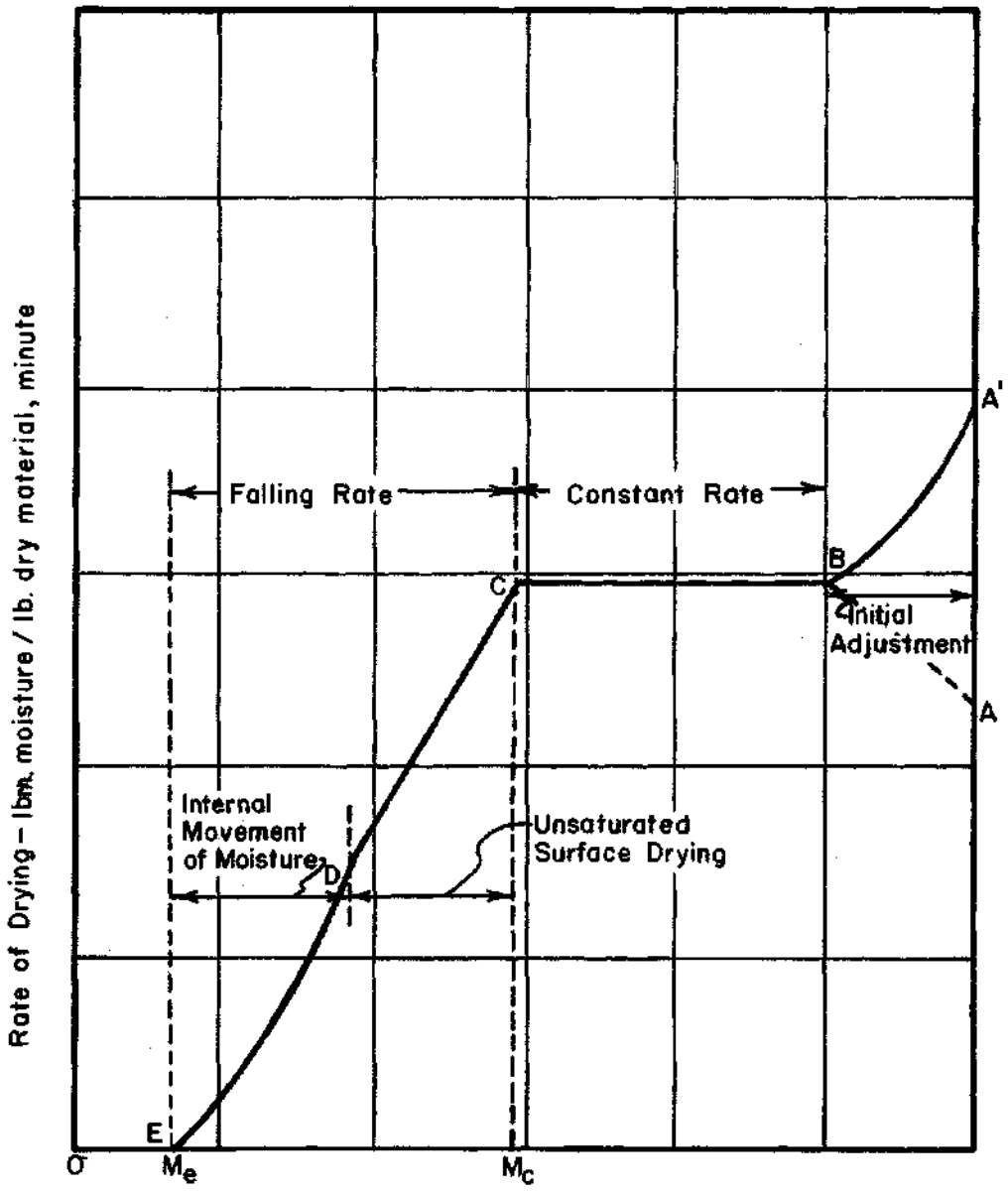


Figure 2. Typical Rate of Drying Curve.

is great, the moisture gradients within the substance will be steep and the average moisture content will be considerably greater than that at the surface. For this reason the critical moisture content (average through the material) increases with an increase in the rate of drying and with an increase in the thickness of substance being dried.[3]

Below the critical moisture content, portion CD, Figure 2, the surface film begins to disappear. This is the first part of the falling rate period and is referred to as unsaturated surface drying. Upon reaching point D, the surface moisture is removed. In certain drying operations the portion CD of the curve is completely missing.

At point D, the second portion of the local falling rate period begins. In this region the rate of drying is dependent on the rate of diffusion of moisture from within the body to the surface. This is the least understood and most complex region of drying. The drying rate in this period varies greatly with the different substances being dried.

The drying processes occurring in the through-flow drying of thick beds can be best understood by considering a series of very thin beds similar to that described in the preceding paragraphs. The process is complicated by the varying free stream conditions which result from the drying process as the air moves through the bed. This causes the shape of the rate of drying curves occurring at different sections to be different.[4] If a bed of material being dried by air passing through the bed is divided into three sections, the rate of drying curves corresponding to the different sections will be similar to those shown in Figure 3. Notice that the entire curves corresponding to the three different sections are deflected from the curve corresponding to the inlet of the bed. These

curves represent the rate of drying under constant drying conditions since the free stream air temperature and concentration are constant at different locations in the bed. As the air progresses through the bed its temperature decreases and concentration increases; therefore, the sections of the bed downstream are exposed to a lower temperature and higher concentration. Thus, the drying rates associated with these sections are lower than the drying rates occurring at the inlet of the bed. After the moisture at the inlet section of the bed is reduced below the critical point, the reduction in temperature and increase in concentration is not as great as during the initial adjustment and constant rate periods. Therefore, the air temperature and concentration to which the sections 2 and 3 in Figure 3 are exposed is higher than when the inlet section was in the constant rate period and; hence, the rate of drying during the constant rate period for sections 2 and 3 will increase as shown in Figure 3.

In experimental measurements an average rate of drying curve results since the entire sample is usually weighed during drying. For thin samples with high porosity the average drying curve will deviate very little from a curve occurring under constant free stream conditions.

Review of Related Literature

In this dissertation the discussion will be restricted to information directly related to through-flow drying problems and drying of related agricultural products.

The first known attempt at drying hay was in Louisiana in about 1911, the exact time and place are not known. The story relates that an

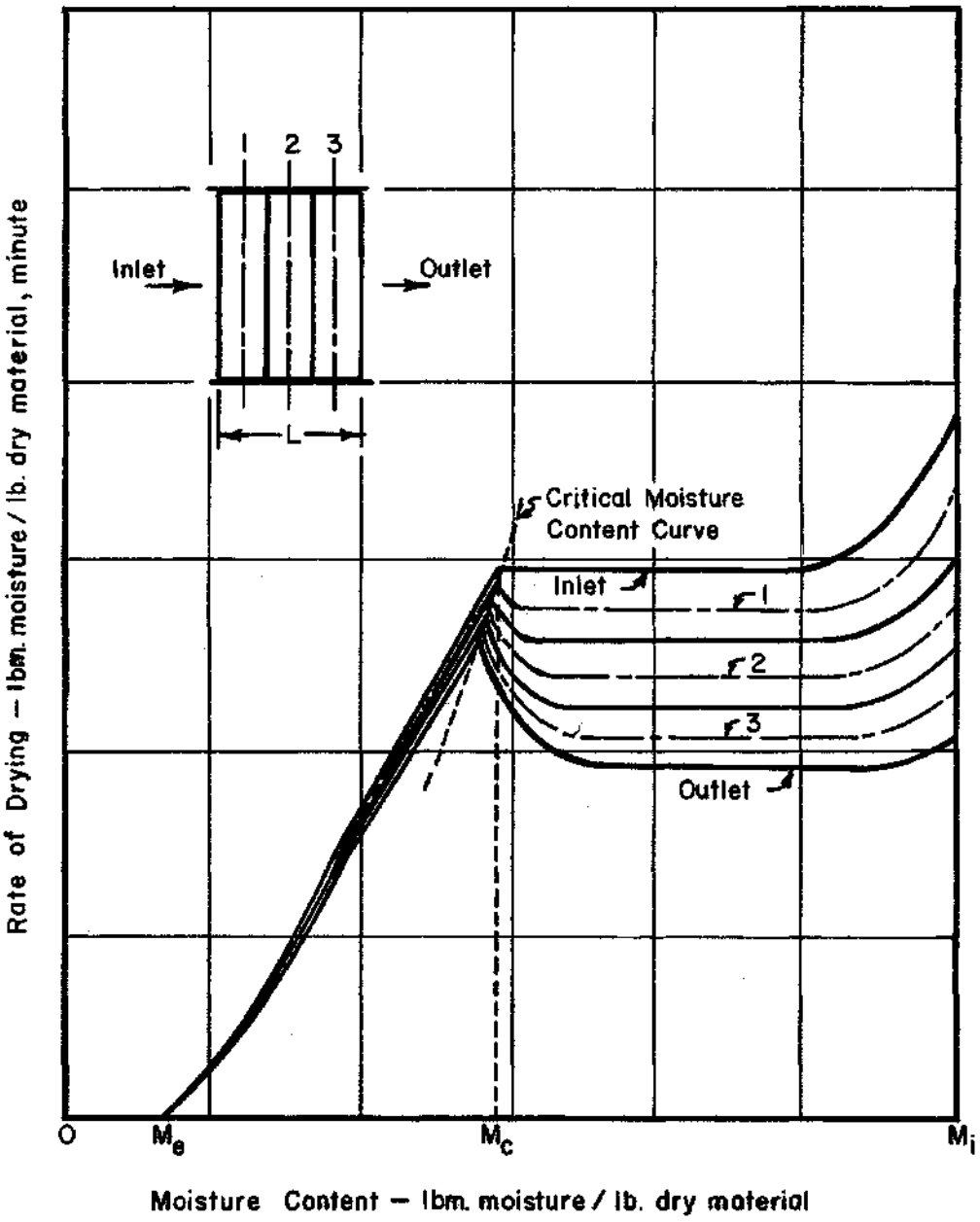


Figure 3. Typical Rate of Drying Curve for Bed.

ingenious agricultural student placed a direct flame under a bed of hay. The material dried quite rapidly and then caught fire. The embarassed student wrote the incident off as a poor idea.[5] Recorded studies on artificial drying of grass were observed as early as 1928[6] in England and by the late 1930's, various crude drying installations were operating there.[7] In 1944 Watson,[1] Fairclough,[8] and Goldberg and Bartelli[9] presented a very complete set of articles on the progress which had been made in grass drying in England. After the war in the middle and late 1940's, interest in slow air drying of hay in storage flourished in the United States.[8,10,11,12,13] This slow method of drying was replaced by artificial drum-type driers in the early 1950's.

Very little analytical work relating to the mechanism of drying has been reported on the subject of hay drying. Most of the articles on this subject relate to only the experimental phase of the process.

Fluid flow through beds of solids and banks of tubes has been analyzed by a number of authorities. However, the results are of little value in hay drying because they involve quantities that are difficult or impossible to determine for hay. Bakhmeteff[15] presented a theory for flow over a collection of spheres immersed in a moving fluid. Guillou[16] improved on this approach and arrived at a very simple expression for the pressure drop as a function of velocity. Guillou's relation serves as a good method of correlation for variations in bed thickness and velocity at given temperatures. For temperature variations, however, the correlation breaks down. McEwen[17] presented a very complete experimental study of resistance to air flow of grain along with a series of previously used correlations for calculating the pressure drop across packed beds.

These correlations can be applied to hay if the particle size and shape can be estimated. Unfortunately most of the published work directly related to flow through beds of hay other than Guillou's was concerned with very low flow rates through very thick beds.

Analytical expressions for the rate of drying during the constant rate period can be obtained by assuming thermodynamic equilibrium in the presence of liquid water exists at the drying interface. Various different approaches can be taken depending on the type of drying and material being dried. One such presentation for this period of through-flow drying was made by Allerton, Brownell, and Katz.[18] They presented a study of the basic mechanism for through-flow drying of filter cakes. They assumed that the air left the bed saturated with water at the adiabatic saturation temperature and picked up moisture in the bed so rapidly that drying could be considered to take place in a narrow zone which gradually moved through the wet bed. Their approach required the experimental measurements of a group of terms which controlled the drying rates. Treybal[19] presented a more generalized approach in a more concise form. Various new approaches have been presented for the unbound region but none deviate greatly from the work by Treybal.

Studies of the highly complex falling rate period are very scattered and limited. Investigations in the region must be restricted to particular materials of interest since no general technique for predicting drying rates during this region has been presented. Although no articles on drying hay in this region were found, for completeness, a number of the articles on the through-flow drying of different materials in the falling rate period will be mentioned since the theories set forth

may be applicable in restricted cases to hay drying. Cassie and Baxter[20] have investigated the process of temperature and moisture propagation through beds of fibers. Their work was concerned with low speed flow and material with low moisture content. McMahon and Downes[21] took into account the finite rate at which equilibrium is established; however, they used an approximation which required the experimental measurement of a rate constant that varies greatly with different materials. Much work has been done on the drying of porous, thick materials,[21,22,23,24] but none of these seem applicable to the extremely thin particle size occurring in hay drying.

Many approximate analytical solutions for the falling rate period have been presented. Perry[3] suggested an approach which would be applicable to essentially any material. In this approach, the falling rate period was approximated as a straight line between the critical moisture content and the equilibrium moisture content. Walker[25] suggested a slightly more complicated approach which applies when the drying conditions are such that the water diffuses to the surface before evaporating. He assumed the concentration distribution within the material at the beginning of the falling rate period. However, this method also requires information on the diffusivity of the water-material mixture which is often not available.

A theoretical method of determining the critical moisture content could not be found; however, Perry[3] does give a qualitative explanation of how this value varies with the conditions and types of material being dried.

A detailed discussion of the equilibrium moisture content will not

be given here since in the range of temperatures in the tests run in the present investigation the concentration of water vapor in the air was so low that the final moisture content of the material was almost zero. Henderson[26] has studied this problem analytically and obtained an equation which allows a qualitative explanation to this problem; however, his equation requires the use of an experimental, measured constants. Dexter, Sheldon, and Waldron[27] presented an experimental method of obtaining the equilibrium moisture content of hay. McEwen,[17] et al., gives three different methods of determining the equilibrium moisture content. Regardless of the experimental method used the time period required to obtain each data point is normally very long.

No articles covering the initial adjustment period could be found concerning any material. The theory related to this period in the through-flow drying of hay will be treated in detail in Chapter III.

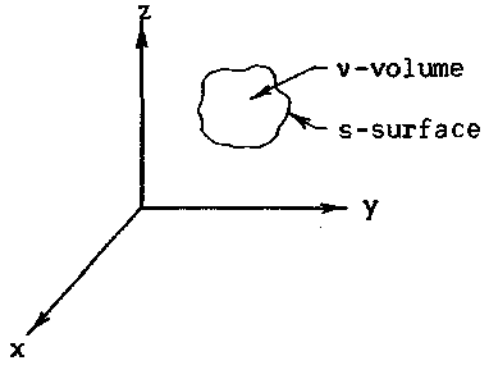
CHAPTER III

THEORY

The determination of the rates of convective drying in general requires equations relating the conditions in the free stream to the surface and knowledge of the relationship between the temperature and concentration at the surface. In this chapter the theoretical equations will be derived in a general form.

Gas Phase Derivation

The continuity equation for the air-water vapor mixture flowing through the bed will be derived by considering the arbitrary control volume shown below. The control volume will consist of an arbitrary region in space. The mass entering from the wet material in the control volume will be considered as mass generated and the heat transfer to the hay will be considered as negative heat generation.



The law of conservation of mass may be written as:

$$\left\{\begin{matrix}\text{rate of}\\ \text{mass flow}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass flow}\\ \text{out}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of}\\ \text{generation}\\ \text{of mass}\end{matrix}\right\} = \left\{\begin{matrix}\text{time rate of change}\\ \text{of mass inside control}\\ \text{volume in gas phase}\end{matrix}\right\}$$

or

$$-\int_s \bar{N}_i \bar{n}_o ds + \int_v R_i dv = \frac{\partial}{\partial \tau}\left[\int_v c_i dv\right]$$

Where: N_i = molar flux of species i, lb mole of i/min ft^2

c_i = apparent molar concentration of species i, lb mole of i/ft^3

R_i = molar rate of mass generation per unit volume, moles of i/ft^3 minute

n_o = direction cosine of unit normal

τ = time

Using the Gauss Divergence Theorem we can relate the surface to the volume integral,[28] i.e.,

$$\int_v \nabla \cdot \bar{q}\, dv = \int_s \bar{n}_o \cdot \bar{q}\, ds$$

Therefore:

$$-\int_v \nabla \cdot \bar{N}_i\, dv + \int_v R_i\, dv = \int_v \frac{\partial c_i}{\partial \tau}\, dv$$

Rearranging:

$$\int_v \left[\frac{\partial c_i}{\partial \tau} + \nabla \cdot \bar{N}_i - R_i\right] d\tau = 0$$

Since we are considering an arbitrary control volume, the integrand must be zero; therefore:

$$\frac{\partial c_i}{\partial \tau} + \nabla \cdot \bar{N}_i = R_i \quad i = 1,2,\ldots,n \qquad \ldots\ldots (1)$$

Eq. (1) can also be obtained from the general equations given in Bird, Stewart, and Lightfoot.[29]

A binary mixture of dry air (B) and water vapor (A) flowing in one direction (Z) through the bed of wet material will be considered. The flow rate of air will be assumed constant throughout the bed. By assuming that at the surface of the wet material no air dissolves into the liquid water, the following equation results:

$$R_A = (k_{x,loc}\, a)\, \frac{x_{AS} - x_A}{1 - x_{AS}} \qquad \ldots\ldots (2)$$

Where: a = the interfacial area per unit volume of bed, ft^2/ft^3

$k_{x,loc}$ = local convective mass transfer coefficient, lb-mole/min ft^2

x_A = mole fraction of the water vapor in the mixture at a point in the free stream lb-moles mixture/mole mixture

x_{AS} = average mole fraction of the water vapor at the surface of the wet material, lb-moles moisture/lb-mole mixture

Substituting Eq. (2) into Eq. (1) gives the one-dimensional case

$$\frac{\partial c_A}{\partial \tau} + \frac{\partial N_{AZ}}{\partial Z} = (k_{x,loc}\, a)\, \frac{x_{AS} - x_A}{1 - x_{AS}} \qquad \ldots\ldots (3)$$

Assuming that diffusion is negligible in comparison to the bulk flow term, the relationship between N_{AZ} and N_{BZ} becomes:

$$N_{AZ} = \frac{x_A}{1 - x_A} N_{BZ}$$

Since the air flow is assumed steady:

$$\frac{\partial N_{AZ}}{\partial Z} = N_{BZ} \frac{\partial}{\partial Z}\left(\frac{x_A}{1 - x_A}\right) = N_B\left[\frac{1}{(1-x_A)^2}\right]\frac{\partial x_A}{\partial Z} \qquad \ldots\ (4)$$

Substituting Eq. (4) into Eq. (3) gives:

$$\frac{\partial c_A}{\partial \tau} = (k_{x,loc}\, a)\frac{x_{AS} - x_A}{1 - x_{AS}} - N_B\left[\frac{1}{(1 - x_A)^2}\right]\frac{\partial x_A}{\partial Z} \qquad \ldots\ (5)$$

Now

$$c_A = x_A c(P_\theta)$$

And for an ideal gas:

$$c_A = x_A \frac{P}{RT}(P_\theta) \qquad \ldots\ (6)$$

Where: P = total pressure of mixture, lbf/ft^2

R = universal gas constant, ft lbf/mole ^oR

T = temperature of mixture ^oR

P_θ = porosity of bed, volume of voids/volume of bed

Assuming P constant, $\frac{\partial c_A}{\partial \tau}$ becomes

$$\frac{\partial c_A}{\partial \tau} = \frac{P(P_\theta)}{RT}\left[\frac{\partial x_A}{\partial \tau} - \frac{x_A}{T}\frac{\partial T}{\partial \tau}\right] - \frac{P x_A}{RT}\frac{\partial (P_\theta)}{\partial \tau} \qquad \ldots\ (7)$$

Substituting Eq. (7) into Eq. (5) gives for the gas phase continuity equation:

$$\frac{P(P_\theta)}{RT}\left[\frac{\partial x_A}{\partial \tau} - \frac{x_A}{T}\frac{\partial T}{\partial \tau}\right] - \frac{P x_A}{RT}\frac{\partial (P_\theta)}{\partial \tau} =$$

$$(k_{x,loc}\, a)\frac{x_{AS} - x_A}{1 - x_{AS}} - N_B\left[\frac{1}{(1 - x_A)^2}\right]\frac{\partial x_A}{\partial Z} \qquad \ldots\ (8)$$

The energy equation for the air-water vapor mixture flowing through the bed will be derived by considering the arbitrary control volume as described in the previous section. The law of the conservation of energy related to this volume can be written as:

$$\left\{\begin{array}{c}\text{rate of}\\ \text{accumulation}\\ \text{of}\\ \text{internal energy}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\ \text{enthalpy}\\ \text{in}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\ \text{enthalpy}\\ \text{out}\end{array}\right\} + \left\{\begin{array}{c}\text{rate of}\\ \text{heat}\\ \text{generated}\end{array}\right\}$$

In the above balance heat addition by conduction and radiation, viscous work done on the control volume by the surroundings, changes in elevation and changes in kinetic energy will be neglected. Applying this balance to the control volume gives:

$$\int_s \left(\sum_{i=1}^{n} \bar{N}_i \cdot \bar{H}_i \right) \cdot \bar{n}_o ds - \int_v R_i \cdot \bar{H}_{is} dv + \frac{\partial}{\partial \tau} \int_v \left(\sum_{i=1}^{n} c_i \bar{U}_i \right) dv$$

$$= - \int_v (h_{x,loc}\, a)(T - T_s) dv \qquad \ldots\ (9)$$

Where: $h_{x,loc}$ = convective heat transfer coefficient, BTU/minute, ft^2 ^oR

$\bar{H}_i$ = partial molar enthalpy of species i in the air-water vapor mixture, BTU/lb-mole of i

$\bar{H}_{is}$ = partial molar enthalpy of species i at the drying surface of the wet material, BTU/lb-mole of i

T_s = temperature of the drying surface, ^oR

$\bar{U}_i$ = partial molar internal energy of species i in the air-water vapor mixture, BTU/lb-mole of i

Again using the Gauss Divergence Theorem Eq. (9) becomes:

$$\int_v \nabla \cdot \left(\sum_{i=1}^{n} \bar{N}_i \cdot \bar{H}_i \right) dv - \int_v R_i \bar{H}_{is} dv + \int_v \frac{\partial}{\partial \tau} \left(\sum_{i=1}^{n} c_i \bar{U}_i \right) = \int_v (h_{x,loc}\, a)(T - T_s) dv$$

Or:

$$\int_v \left[\frac{\partial}{\partial \tau} \left(\sum_{i=1}^{n} c_i \bar{U}_i \right) + \nabla \cdot \left(\sum_{i=1}^{n} \bar{N}_i \cdot \bar{H}_i \right) - R_i \bar{H}_{is} - (h_{x,loc}{}^a)(T_s - T) \right] dv = 0$$

Since the control volume is arbitrary, the integrand must be zero; therefore:

$$\frac{\partial}{\partial \tau} \left(\sum_{i=1}^{n} c_i \bar{U}_i \right) + \nabla \cdot \left(\sum_{i=1}^{n} \bar{N}_i \cdot \bar{H}_i \right) - R_i \bar{H}_{is} - (h_{x,loc}{}^a)(T_s - T) = 0 \quad ..(10)$$

Adding $\frac{\partial P}{\partial \tau}$ to each side Eq. (11) becomes:

$$\frac{\partial}{\partial \tau} \left(\sum_{i=1}^{n} c_i \bar{H}_i \right) + \nabla \cdot \left(\sum_{i=1}^{n} \bar{N}_i \cdot \bar{H}_i \right) - R_i \bar{H}_{is} - (h_{x,loc}{}^a)(T - T_s) = \frac{\partial P}{\partial \tau} \quad ...(11)$$

For a binary mixture of air (B) and water vapor (A) flowing in one direction (Z) through the bed of wet material Eq. (11) becomes:

$$\frac{\partial}{\partial \tau} (c_A \bar{H}_A) + \frac{\partial}{\partial \tau} (c_B \bar{H}_B) + \frac{\partial}{\partial Z} (N_A \bar{H}_A) + \frac{\partial}{\partial Z} (N_B \bar{H}_B)$$

$$- R_A \bar{H}_{AS} - (h_{x,loc}{}^a)(T_s - T) = \frac{\partial P}{\partial \tau} \quad .... (12)$$

Which can be written as:

$$\bar{H}_A \left[\frac{\partial c_A}{\partial \tau} + \frac{\partial N_A}{\partial Z} \right] + \bar{H}_B \left[\frac{\partial c_B}{\partial \tau} + \frac{\partial N_B}{\partial Z} \right] + c_A \frac{\partial \bar{H}_A}{\partial \tau} + c_B \frac{\partial \bar{H}_B}{\partial \tau}$$

$$N_A \frac{\partial \bar{H}_A}{\partial Z} + N_B \frac{\partial \bar{H}_B}{\partial Z} - R_A H_{AS} = (h_{x,loc}{}^a)(T_s - T) + \frac{\partial P}{\partial \tau} \quad ....(13)$$

The term in the second group of brackets is zero by continuity since the flux of air is zero at the surface of the wet material. Thus by substituting Eq. (3) and Eq. (4), Eq. (13) becomes:

$$(k_{x,loc}{}^{a})\,\frac{x_{AS} - x_A}{1 - x_{AS}}\left[\tilde{C}_{PA}(T - T_s)\right] + c_A\tilde{C}_{PA}\,\frac{\partial T}{\partial \tau} + c_B\tilde{C}_{PB}\,\frac{\partial T}{\partial \tau}$$

$$+ N_A\tilde{C}_{PA}\,\frac{\partial T}{\partial Z} + N_B\,\tilde{C}_{PB}\,\frac{\partial T}{\partial Z} = (h_{x,loc}{}^{a})(T_s - T) + \frac{\partial P}{\partial \tau} \qquad \ldots. \text{(14a)}$$

where: $\tilde{C}_{PA}$ = constant pressure molar specific heat of the water vapor, BTU/lb mole of A °R

$\tilde{C}_{PB}$ = constant pressure molar specific heat of air, BTU/lb mole of B °R

Substituting Eq. (7) into Eq. (13) gives:

$$\frac{P(P_o)}{RT}\left[x_A\tilde{C}_{PA} + x_B\tilde{C}_{PB}\right]\frac{\partial T}{\partial \tau} + \left[N_A\tilde{C}_{PA} + N_B\tilde{C}_{PB}\right]\frac{\partial T}{\partial Z}$$

$$= (k_{x,loc}{}^{a})(T_s - T)\left[\frac{(h_{x,loc}{}^{a})}{(k_{x,loc}{}^{a})} + \tilde{C}_{PA}\,\frac{x_{AS} - x_A}{1 - x_{AS}}\right] + \frac{\partial P}{\partial \tau} \quad \ldots\text{(14b)}$$

Again from Eq. (4) neglecting the diffusion term gives:

$$\frac{N_{AZ}}{N_{BZ}} = \frac{X_A}{1 - X_A}$$

Substituting into Eq. (14b) gives:

$$\frac{P(P_o)}{RT}\left[x_A(\tilde{C}_{PA} - \tilde{C}_{PB}) + \tilde{C}_{PB}\right]\frac{\partial T}{\partial \tau} + \frac{N_B}{1 - x_A}\left[x_A(\tilde{C}_{PA} - \tilde{C}_{PB}) + \tilde{C}_{PB}\right]\frac{\partial T}{\partial Z}$$

$$= (k_{x,loc}{}^{a})(T_s - T)\left[\frac{(h_{x,loc}{}^{a})}{(k_{x,loc}{}^{a})} + \tilde{C}_{PA}\,\frac{x_{AS} - x_A}{1 - x_{AS}}\right] + \frac{\partial P}{\partial \tau} \qquad \ldots \text{(15)}$$

Applying the analogy between heat and mass transfer gives:

$$\frac{(h_{x,loc}a)}{C_p G}(Pr)^{2/3} = \frac{(k_{x,loc}a)\rho}{cG}(Sc)^{2/3}$$

Where: Pr = Prandtl number

Sc = Schmidt number

Since:

$$c\tilde{C}_p = \rho C_p$$

Thus:

$$\frac{(h_{x,loc}a)}{(k_{x,loc}a)} = \tilde{C}_p\left[\frac{Sc}{Pr}\right]^{2/3} \quad \ldots\ (16)$$

Thus Eq. (15) becomes:

$$\frac{P(P_o)}{RT}\left[x_A(\tilde{C}_{PA} - \tilde{C}_{PB}) + \tilde{C}_{PB}\right]\frac{\partial T}{\partial \tau} + \frac{N_B}{1-x_A}\left[x_A(\tilde{C}_{PA} - \tilde{C}_{PB}) + \tilde{C}_{PB}\right]\frac{\partial T}{\partial Z}$$

$$= (k_{x,loc}a)(T_s - T)\left\{\left[x_A(\tilde{C}_{PA} - \tilde{C}_{PB}) + \tilde{C}_{PB}\right]\left[\frac{Sc}{Pr}\right]^{2/3} + \tilde{C}_{PA}\frac{x_{AS} - x_A}{1 - x_{AS}}\right\}$$

$$+ \frac{\partial P}{\partial \tau} \quad \ldots\ (17)$$

Material Phase Derivation

The continuity equation for the wet material will be derived by considering an arbitrary control volume in space. The mass leaving the wet material will be considered as a negative mass generation term and the heat transfer to the wet bed will be considered as a heat generation

term. The conservation of mass gives:

$$\left\{\begin{matrix}\text{rate of}\\ \text{mass flow}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass flow}\\ \text{out}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of mass}\\ \text{generation}\end{matrix}\right\} = \left\{\begin{matrix}\text{time rate}\\ \text{of change of}\\ \text{mass}\end{matrix}\right\}$$

Thus since there is no flux of mass in or out of the control volume and since the mass entering the control volume from the wet material is considered as mass generated, the conservation of mass reduces to:

$$\frac{\partial}{\partial \tau}\left[\int_v c_j dv\right] = \int_v R_j dv$$

or:

$$\int_v \left[\frac{\partial c_j}{\partial \tau} - R_j\right] dv = 0$$

Now applying the above to the mixture of water and dry material and noting that $R_i = -R_j$ since the mass entering the gas phase must equal the mass leaving the material phase:

$$\int_v \left[\frac{\partial C_{A\ell}}{\partial \tau} + R_A\right] dv = 0 \qquad \ldots. (18)$$

Where: $C_{A\ell}$ = apparent molar concentration of water in the volume, moles of A/ft.^{3}

Now for an arbitrary control volume:

$$\frac{\partial C_{A\ell}}{\partial \tau} + R_A = 0 \qquad \ldots. (19)$$

Since $M_A C_{A\ell} = \frac{m_w}{V}$

And: $$M = \frac{m_w}{m_h} = \frac{M_A C_{A\ell} V}{m_h}$$

Where: M_A = molecular weight of water, lbm/mole

Therefore: Eq. (19) becomes:

$$\frac{\partial M}{\partial \tau} = (k_{x,loc}a)\frac{M_A V}{m_h}\frac{x_{AS} - x_A}{1 - x_{AS}} \qquad ...(20)$$

The energy equation for the wet material will be derived by considering the arbitrary control volume of material as described in the previous section. The conservation of energy is:

$$\left\{\begin{matrix}\text{rate of}\\ \text{accumulation}\\ \text{of internal}\\ \text{energy}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{enthalpy}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{enthalpy}\\ \text{out}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of}\\ \text{heat generation}\end{matrix}\right\}$$

In the above balance heat addition by conduction and radiation, viscous work done on the control volume by the surroundings, changes in elevation and changes in kinetic energy have been neglected.

Thus:

$$-\int_v R_j \bar{H}_{js} dv + \frac{\partial}{\partial \tau}\int_v \left(\sum_{j=1}^{n} c_j \bar{U}_j\right) dv = -\int_v (h_{x,loc}a)(T_s - T)dv$$

Or:

$$\int_v \left[\frac{\partial}{\partial \tau}\left(\sum_{j=1}^{n} c_j \bar{U}_j\right) - R_j \bar{H}_{js} + (h_{x,loc}a)(T_s - T)\right] dv = 0 \qquad ....(21)$$

Assuming the control volume is arbitrary and that the first term

in Eq. (21) can be separated, Eq. (21) can be reduced. This will introduce a slight error since a chemical reaction occurs during drying of hay which results in a generation of heat; however, the error should be small since the magnitude of the heat generated is small when compared to the heat of vaporization of water.

$$\frac{\partial}{\partial \tau}\left[C_{A\ell}U_{A\ell} + c_h U_h\right] + R_A \bar{H}_{AS} = (h_{x,loc}{}^a)(T_s - T) \qquad \ldots(21a)$$

Where: c_h = apparent molar concentration of dry material, moles/ft^3

$U_{A\ell}$ = molar internal energy of the liquid water, BTU/lb-mole

U_h = molar internal energy of dry material, BTU/lb-mole

Thus Eq. (21a) can be written as:

$$U_{A\ell}\frac{\partial c_{A\ell}}{\partial \tau} + C_{A\ell}\frac{\partial U_{A\ell}}{\partial \tau} + c_h\frac{\partial U_h}{\partial \tau} + R_A\bar{H}_{AS} = -(h_{x,loc}{}^a)(T_s - T) \qquad \ldots(22)$$

From Eq. (19)

$$\frac{\partial C_{A\ell}}{\partial \tau} = -R_A$$

Thus:

$$\left[C_{A\ell}\tilde{C}_{vA\ell}\frac{\partial T_s}{\partial \tau} + c_h\tilde{C}_{vh}\frac{\partial T_s}{\partial \tau}\right] = (h_{x,loc}{}^a)(T - T_s) - R_A(\bar{H}_{AS} - U_{A\ell}) \quad ..(23)$$

Where: $\tilde{C}_{vA\ell}$ = constant volume molar specific heat of liquid water, BTU/lb-mole ^oR

$\tilde{C}_{vh}$ = constant volume molar specific heat of dry hay, BTU/lb-mole ^oR

Assuming: $\bar{H}_{AS} - U_{A\ell} = H_{fg}$

Where: H_{fg} = molar heat of vaporization at the surface temperature, BTU/lb-mole

Substituting Eq. (2) and Eq. (16) into Eq. (23) gives:

$$\left[c_{A\ell}\tilde{C}_{vA\ell}\frac{\partial T_s}{\partial \tau} + c_h\tilde{C}_{vh}\frac{\partial T_s}{\partial \tau}\right] = (k_{x,loc}a)\left\{\left[x_A(\tilde{C}_{PA} - \tilde{C}_{PB}) + \tilde{C}_{PB}\right]\left[\frac{Sc}{Pr}\right]^{2/3}(T - T_s) + \frac{x_{AS} - x_A}{1 - x_{AS}}H_{fg}\right\} \quad \ldots\ (24)$$

A more convenient form of Eq. (24) can be obtained by writing the first term in Eq. (24) in terms of lbm instead of moles. Thus:

$$\left[(m_h c_{ph} + m_w c_{pw})\frac{1}{V}\right]\frac{\partial T_s}{\partial \tau} = (k_{x,loc}a)\left\{\left[x_A(\tilde{C}_{pA} - \tilde{C}_{pB}) + \tilde{C}_{pB}\right]\left[\frac{Sc}{Pr}\right]^{2/3}(T - T_s) - \frac{x_{AS} - x_A}{1 - x_{AS}}H_{fg}\right\} \quad \ldots\ (25)$$

Where: c_{ph} = specific heat of dry material, BTU/lbm °R

c_{pw} = specific heat of liquid water, BTU/lbm °R

The simultaneous solution of Eqs. (3), (17), (20) and (25) along with a relationship between the temperature and concentration at the surface of the drying material and information about the mass transfer coefficient enables the determination of the drying rate. The solution of these equations is given in Appendix A.

CHAPTER IV

EXPERIMENTAL EQUIPMENT

In the present investigation three separate experimental apparatuses were used. Diagrammatic sketches and photographs of the equipment are shown in Figures 4 through 9, and they will be discussed in the following paragraphs.

Wind-Tunnel Drier

The wind-tunnel drier used in this investigation is shown in a diagrammatic sketch in Figure 4 and by a photograph in Figure 5. The basic apparatus was constructed and first used by Brock.[30] The components of the drier can best be explained by tracing the flow of air through the unit. Air enters the drier through an orifice plate and travels through a straightening section into the inlet of a New York Blower No. 152, backward-curved blade blower. The air flow rate through the drier is measured by the inlet orifice plate. The air leaves the blower and travels through an opposed blade outlet damper and then through a flexible rubber section between the damper and a transition section leading to three sets of finned tubes. The air is heated by high pressure steam in the finned tubes. The air then travels into a straightening duct leading to the 12 inch diameter test section. In the test section the air travels through the sample and then is exhausted from the drier.

The pressure drop across the orifice plate was measured by a

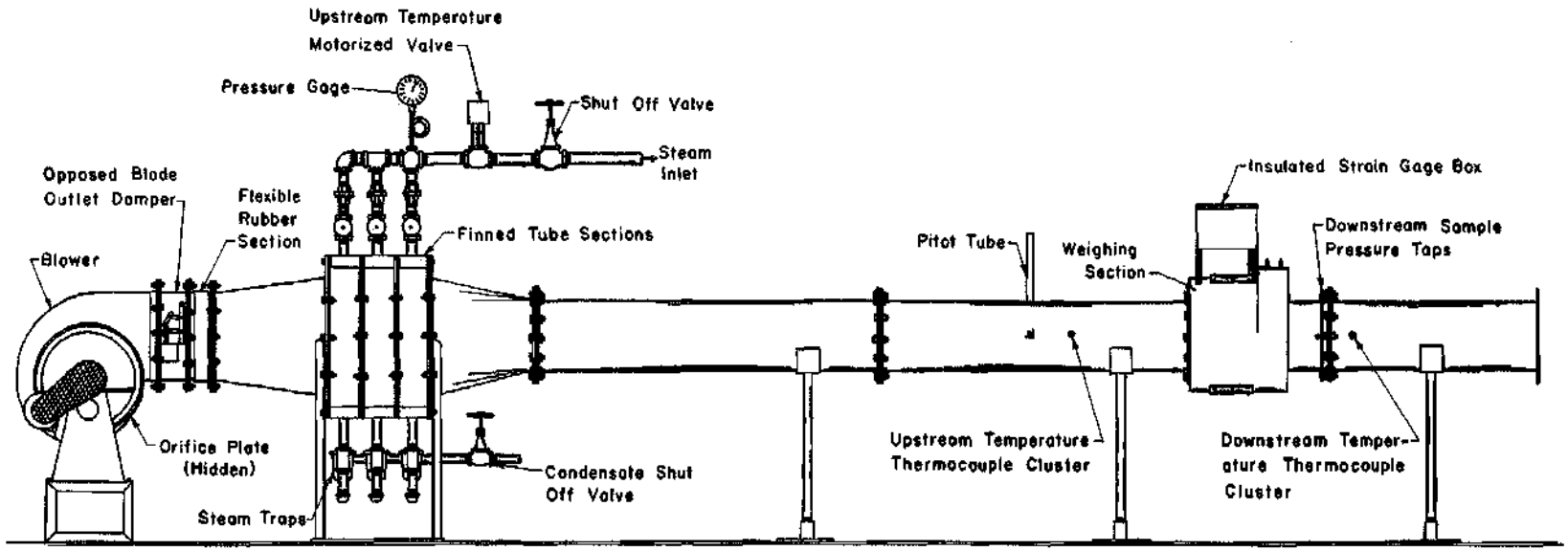


Figure 4. Schematic Diagram of Wind-Tunnel Drier.

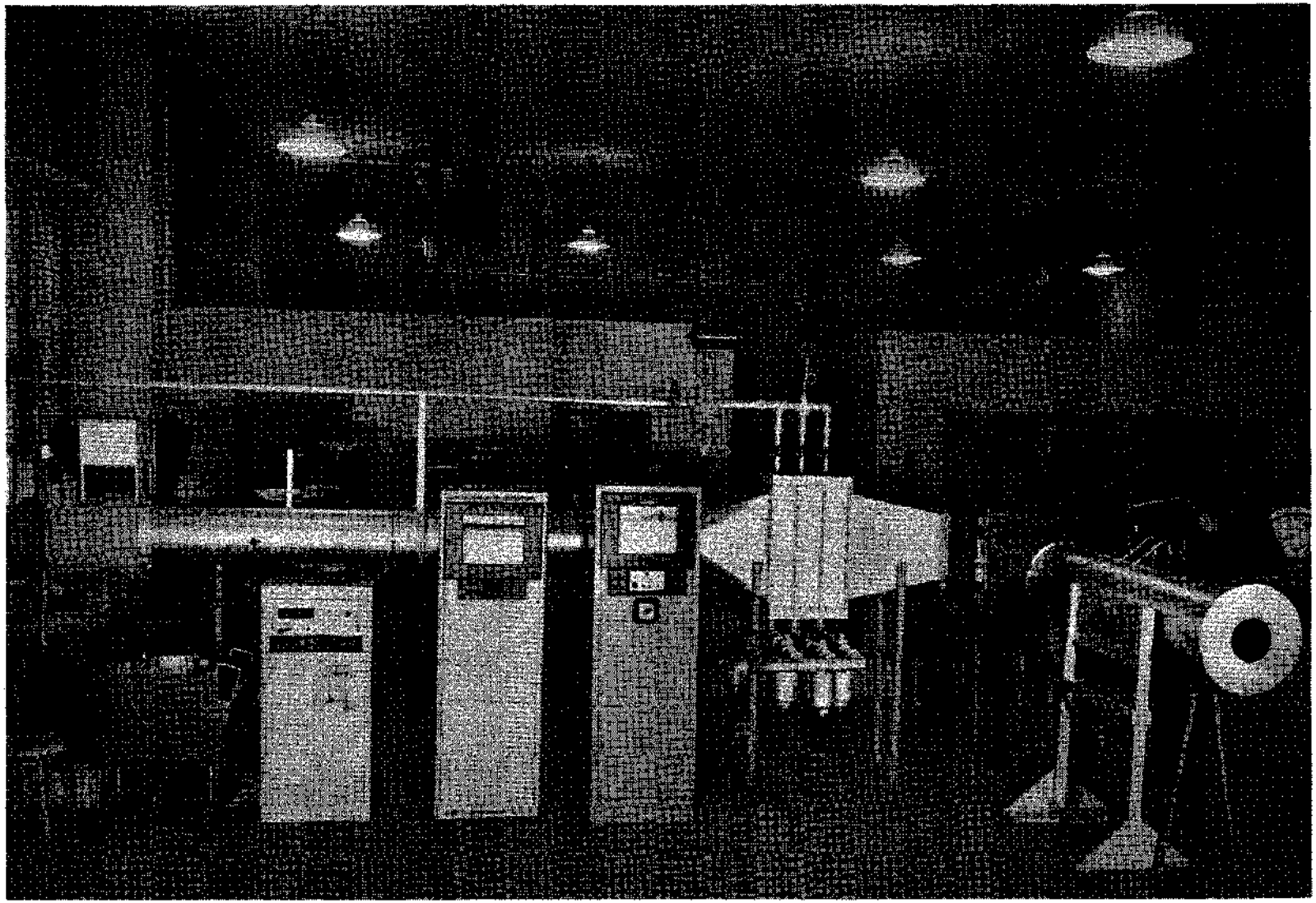


Figure 5. Photograph of Wind-Tunnel Drier.

2 inch inclined manometer connected to a manifold down-stream of the orifice plate. The orifice plate was calibrated by ten 20 point pitot tube traverses. The pressure drop across the sample was measured in inches of water by a six inch Trimount micro-manometer. Manifolds with four pressure pickups around the tunnel upstream and downstream were connected to the manometer. For a number of runs the inclined manometer at the orifice plate and the 6 inch micro-manometer were placed by a pressure transducer connected to one channel of a Sanborn 150 strain gauge amplifier. This gave a continuous plot of either variation in flow rate or pressure drop variation through the sample at a constant flow rate.

A schematic diagram of the control, measuring and recording system is shown in Figure 6. The wet and dry bulb temperatures of the free stream air at the entrance to the drier and the dry bulb temperature downstream of the orifice plate were measured by mercury in glass thermometers. The temperature of the heated air was measured upstream of the test section by three chromel-alumel thermocouples. This temperature was controlled by a Minneapolis-Honeywell Three-Mode Electro-O-Line Control Unit and Electronik recorder operating with a model 1½-800 Minneapolis-Honeywell motorized valve in the steam supply line leading to the Steelfin tube sections. Both the wet and dry bulb temperature of the free stream air downstream of the sample were measured. Again three chromel-alumel thermocouples were connected to a Minneapolis-Honeywell Electronik recorder.

The weighing section is illustrated diagrammatically in Figure 7, and by a photograph in Figure 8. The weight change of the sample during drying was determined by the change in deflection of six one-sixteenth

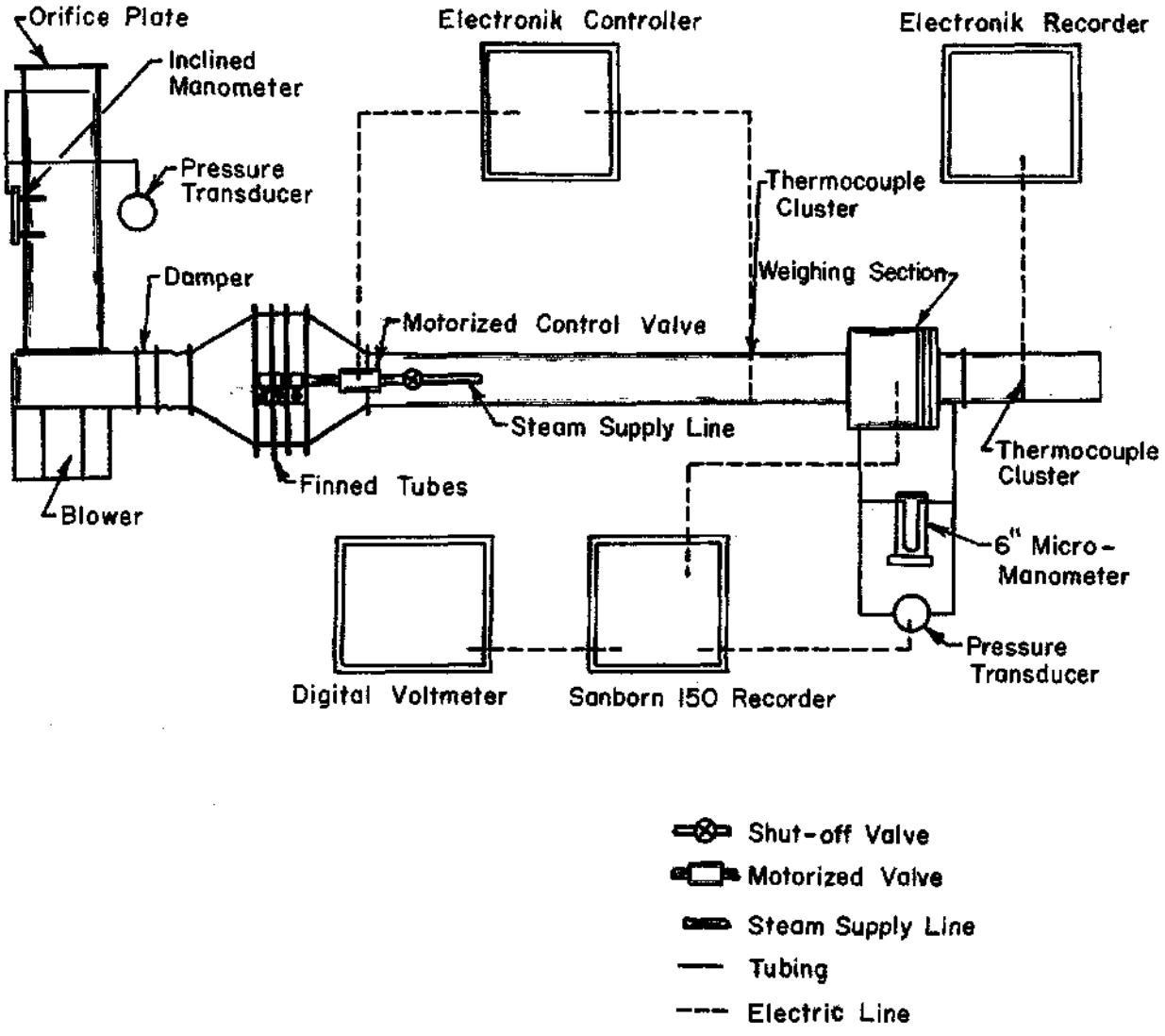


Figure 6. Schematic Diagram of Control, Measuring and Recording System

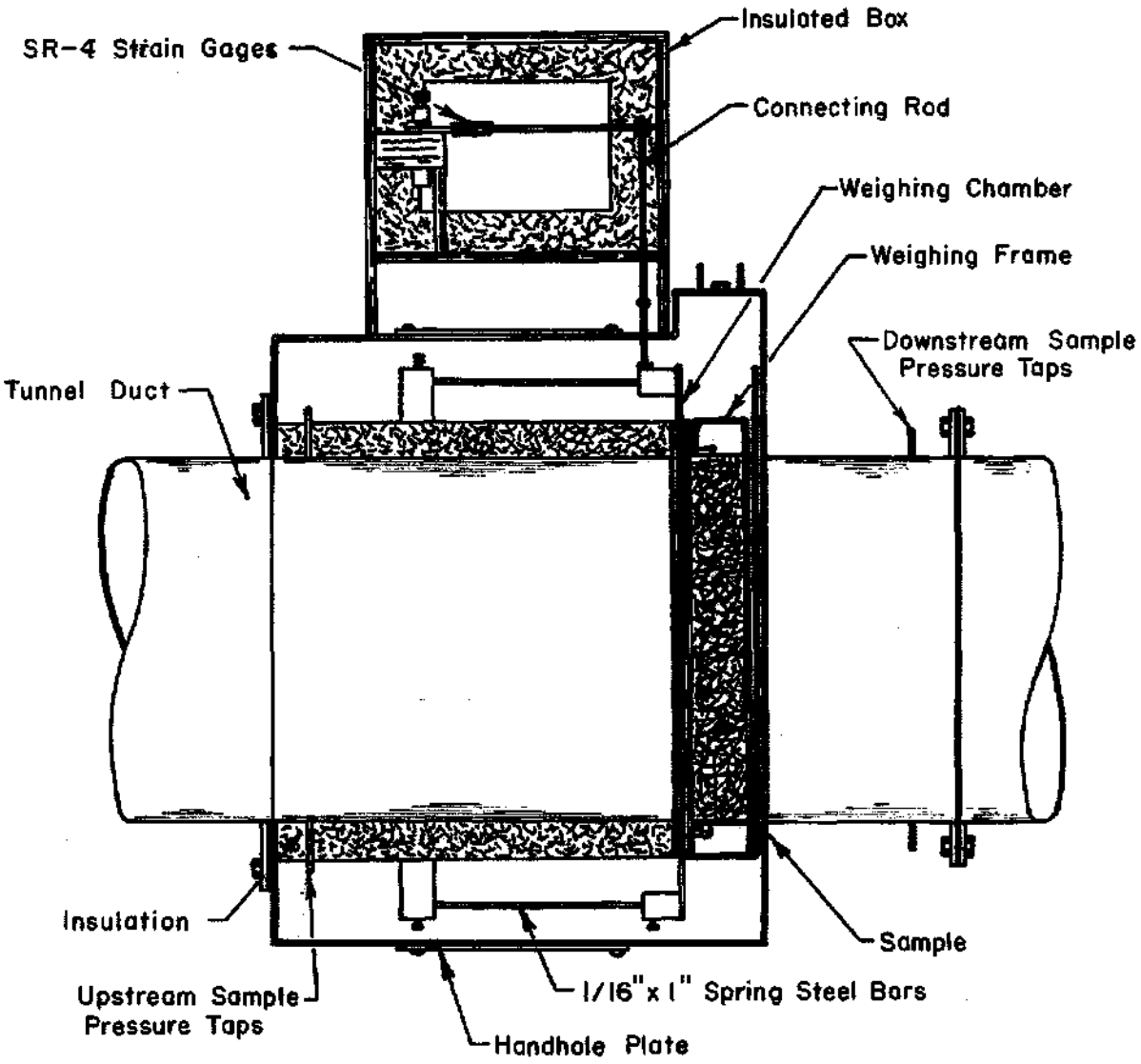


Figure 7. Schematic Diagram of Weighing Section

by one inch spring steel bars. The weighing section frame was attached to four of the bars which acted as cantilevered beams. The other two bars were mounted above the weighing section in an insulated box. Four Baldwin-Lima-Hamilton SR-4 strain gauges were assembled on these bars. The two bars were connected to the other four bars by a rod running through the bottom of the box. The insulation in the box reduces temperature and humidity changes around the strain gauges thus reducing drift. A full strain gage bridge was connected to a Sanborn 150 strain gage amplifier. Output from the amplifier was connected to a Hewlett-Packard Automatic DC digital voltmeter which gave a print-out of volts versus time giving the drying curve. The weighing section has a maximum capacity of 30 pounds, and it has a sensitivity of 0.002 pounds. The system was calibrated using known weights. Before and after each run the sample was weighed on a nonspring Toledo balance scale to check the weight change indicated on the digital voltmeter.

One of the sample frames is shown in Figure 8. The four frames permitted a variation in bed thickness of from one to four inches in increments of one inch. The front of each frame was hinged acting as a door to allow the hay to be placed in the frame. Thirty mesh stainless steel wire was used on the front and back of each frame to support the material. Since the frame was vertical during drying, as shown in Figure 7, page 34, three horizontal pieces of hardware cloth were placed in the frame to prevent the hay from settling which would cause non-uniform air flow through the bed.

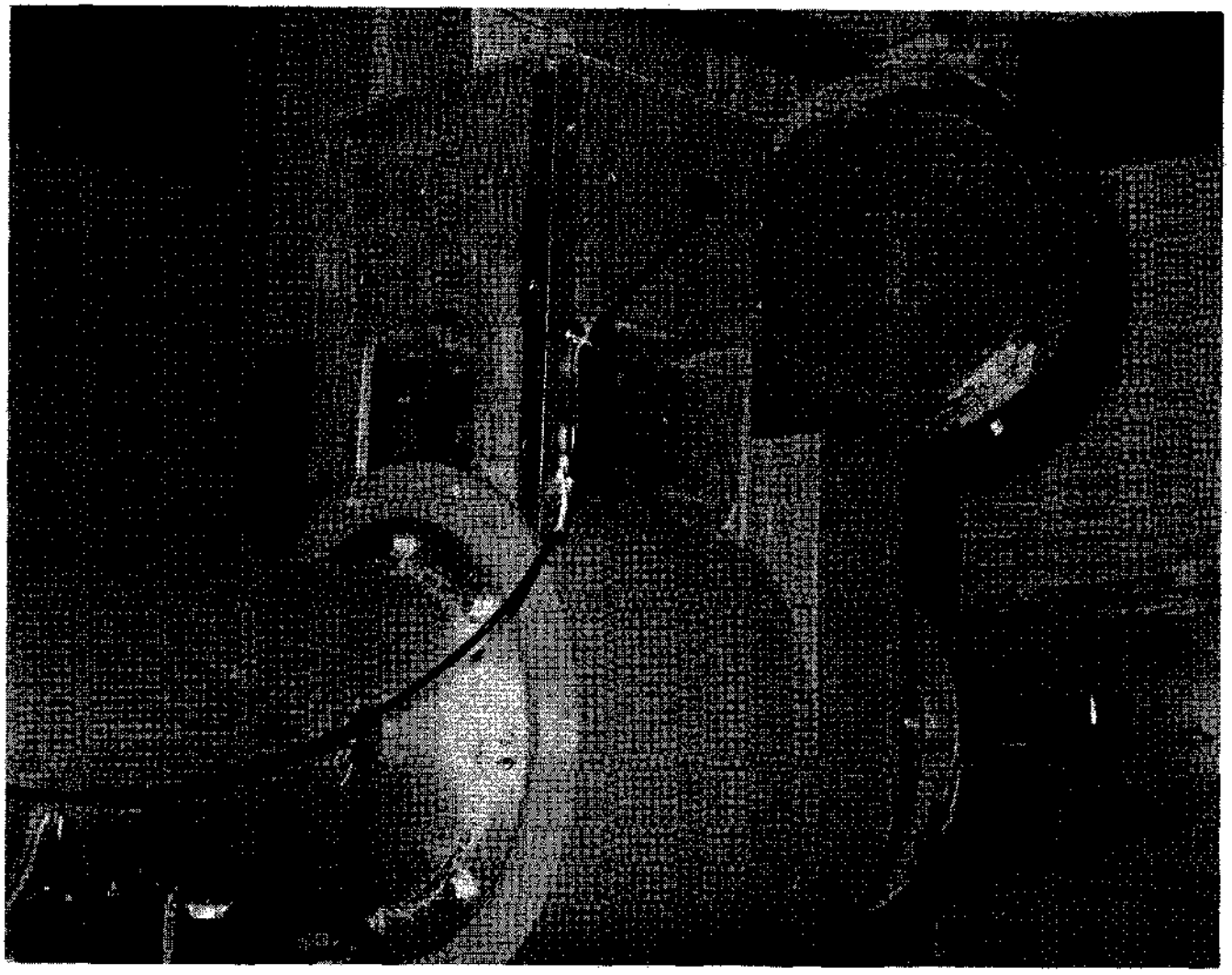


Figure 8. Photograph of Weighing Section.

Vacuum Chamber

A standard vacuum chamber was used in this investigation. This unit was used to obtain the dry weight of each sample. A detailed discussion of this apparatus is given in the thesis by Hardin.[31] A Welch Scientific Vacuum pump was used to evacuate the chamber. The chamber pressure was read on a Wallace and Tierman, Model No. FA 160 absolute pressure gauge. The samples were first dried at 180°F or above in the drier and then placed in the chamber for a period of twenty-four hours in order to determine the dry weight.

Porosity Determination System

The device used to measure the volume occupied by the material in the frame is shown in Figure 9. This system was first suggested by Day.[32] The system consists of two tanks of approximately equal volumes and a 36" manometer connected by copper tubing. One tank serves as a plenum and the material under test is placed in the second tank. Air was placed under pressure in the first tank while the second tank was open to the atmosphere. The second tank was then closed and the air contained in the first tank was allowed to flow into the second tank. The pressure of the system was then recorded. Assuming air an ideal gas, the difference in the original pressure and the final pressure is directly proportional to the unknown volume. By placing known volumes in the apparatus a curve of volume versus pressure drop was obtained. This curve was then used in determining the unknown volume. The test procedure is described in Chapter V.

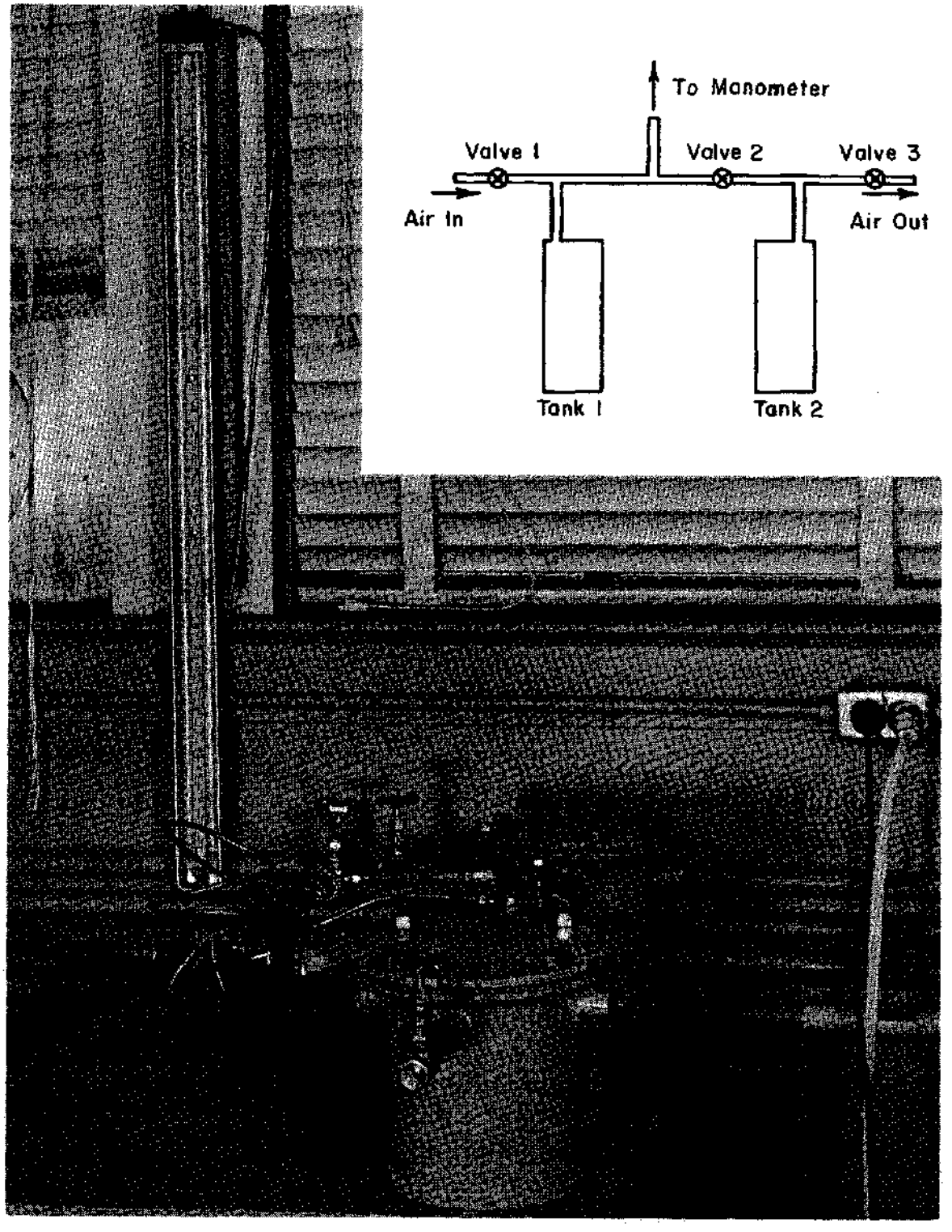


Figure 9. Porosity Determination System.

CHAPTER V

EXPERIMENTAL PROCEDURE

Description of Samples

Over 125 different species of forage grasses and legumes are grown in the United States.[33] Of these species only about ten have widespread usages as hay. In this investigation, a hybrid grass, Coastal Bermuda Grass, was selected since it was felt that it would exhibit drying requirements of most hays. Other hays were tested but not as extensively as coastal bermuda. Coastal bermuda grass was developed at the United States Coastal Plains Experiment Station in Tifton, Georgia. It is a cross between a strain discovered in an old cotton field in Georgia and bermuda grass from South Africa. Since its release in 1939, over 3 million acres have been planted in the South.[34]

The difficulties of dealing with biological materials are evident in the selection of the test material. Hay can only be obtained from the field at harvest time which restricts the period of testing to about four months each year. The material also must be dried within a few hours after cutting if it is to exhibit any of the characteristics of freshly cut field hay. If the material is stored damp it will begin to germinate within a few days. Variations in weather conditions also add difficulties since the moisture content (wet basis) may vary from 50 to 85 per cent in the field. The moisture content of the hay also varies with the time of day and with the age of material.

Preparation of Sample

The test material was hand cut in the field and placed in large bags. The bags were then moved to the lab and the hay was removed and cut into the desired lengths. The cut hay was then placed in the sample frame and weighed. After drying the sample was reweighed and then the dried hay was removed from the frame and placed in a water proof plastic bag ready to be analyzed.

Preparation of Apparatus

The Sanborn strain recorder, digital voltmeter, and Honeywell controllers and recorders operated continuously during the periods of testing. The desired air flow rate was set on the inclined manometer connected to the orifice plate. The desired air temperature was set on the temperature controller and allowed to stabilize. This required a minimum period of thirty minutes.

Test Procedures

After the temperature stabilized, the weighing frame containing hay was placed in the wind tunnel drier. The printer on the digital voltmeter was turned on and the damper was opened until the pre-set flow rate was attained. The damper was continually adjusted to maintain a constant flow rate during the run. During the test, periodic pressure drop readings across the sample were taken. The drier ran until no further change occurred on the digital voltmeter. On a number of runs, the flow rate was allowed to vary through the material as would occur under industrial drying conditions, i.e., the blower speed remained constant and the damper was not adjusted. On these runs a pressure transducer

was connected to the pressure taps on the inlet orifice plate and the flow rate variations were printed on the Sanborn strip chart. Periodic pressure drop readings across the sample were taken. Again the drier was allowed to run until no further change occurred on the digital voltmeter.

On a number of runs the temperature downstream of the sample was measured. The same procedure as in the constant drying runs was made except that the wet and dry bulb temperatures downstream were measured and recorded on a Honeywell strip chart.

The test procedure for determining the porosity of both wet and dry material samples was as follows: (Refer to Figure 9, page 39.)

1. Material was placed in tank No. 2.

2. With valve No. 2 closed, air was supplied to tank No. 1.

3. When a suitable manometer displacement was achieved, valve No. 1 was closed, the manometer was allowed to come to equilibrium and the reading was taken.

4. Valve No. 3 was then closed, valve No. 2 was opened and the manometer again was read, after it came to equilibrium.

After a sample had been dried in the drier at a temperature of 180°F or higher, it was placed in the vacuum chamber at a pressure of from 2 to 5 millimeters of mercury for 24 hours. The sample was then weighed and this value considered to be the dry weight. This weight deviated only slightly from the weight of the sample after drying if the air temperature was above 180°F.

The length of the material was determined by measuring a random number of particles. Care was taken to cut the material in uniform length.

The number of particles in a given sample was determined by counting a given weight of material.

CHAPTER VI

DISCUSSION OF EXPERIMENTAL RESULTS

The results of the experimental studies are shown graphically in Figures 10 through 35. Unless otherwise specified this discussion will be concerned with coastal bermuda grass only. In this investigation, the samples were dried under constant drying conditions; that is, they were exposed to air at a constant temperature, humidity, and velocity. In order to maintain a constant through flow velocity the outlet damper on the blower had to be continuously adjusted. This resulted in a large variation in the pressure drop across the sample as shown in Figure 10. The variation was due mainly to the large volume change occurring in the bed during drying. A series of runs over the range of a particular variable had to be made on each day of testing in order to make accurate deductions, since the initial moisture content (dry basis) of the material varies from 120 per cent to over 450 per cent depending on weather conditions.

Pressure Drop

Perhaps the most unexpected result of this investigation was the wide variation in pressure drop across the sample during drying, indicating a great dependence of the pressure drop on the particle size and, hence indirectly on moisture content of the hay. Figure 10, shows this pressure drop variation for four different bed thicknesses. When the pressure drop across the sample was plotted against the moisture content

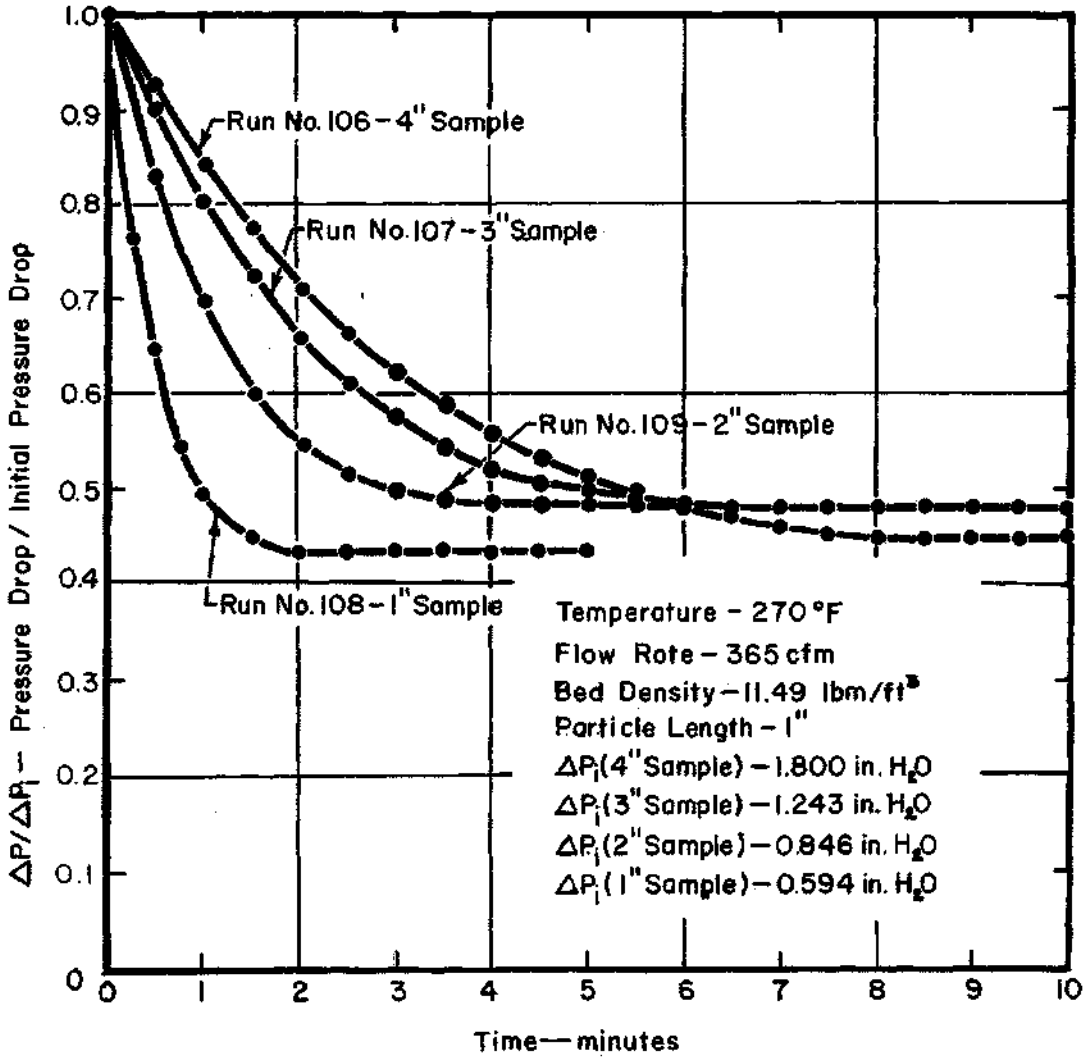


Figure 10. Variation in Pressure Drop across Bed during Drying

in the material, it was found to vary essentially linearly with moisture content between the initial and critical moisture content. Also, each curve appeared to have approximately the same slope. By selecting one curve as a reference, it was found that points for the different runs could be plotted on a single curve when the parameter $Gr = [\text{Equation of curve}]/(\frac{\Delta P}{\Delta P_i})$ was plotted against the parameter M/M_i. The resulting plot is shown in Figure 11.

By using the experimental results obtained from the porosity measuring system described in Chapter IV, it was found that porosity could be determined within the accuracy of the experimental equipment by considering the dry material to be solid, i.e., there are no trapped voids. This result allows the analytical determination of the porosity by Eq. (28). This equation shows the approximate linear variation in moisture content with porosity which was observed experimentally.

In the range of the variables tested, it was found that the pressure drop across the dry sample could be correlated with significant variables by use of a modified Reynolds number and friction factor. The correlation as shown in Figure 12, is scattered; however, the majority of the points fell on the curve. The Reynolds number and friction factor are defined by Eqs. (26) and (27). Figure 11 can be used to extend pressure drop data for a dry sample to a wet sample. The weakness of the correlation seems to be due to the wide variation in porosity and particle length. The very high porosity, 80 to 96 per cent, which occurs in beds of hay is actually out of the range of the correlation, and the method used to determine the particle diameter could not sufficiently adjust for large variations in particle lengths. However, the equations correlated the remaining

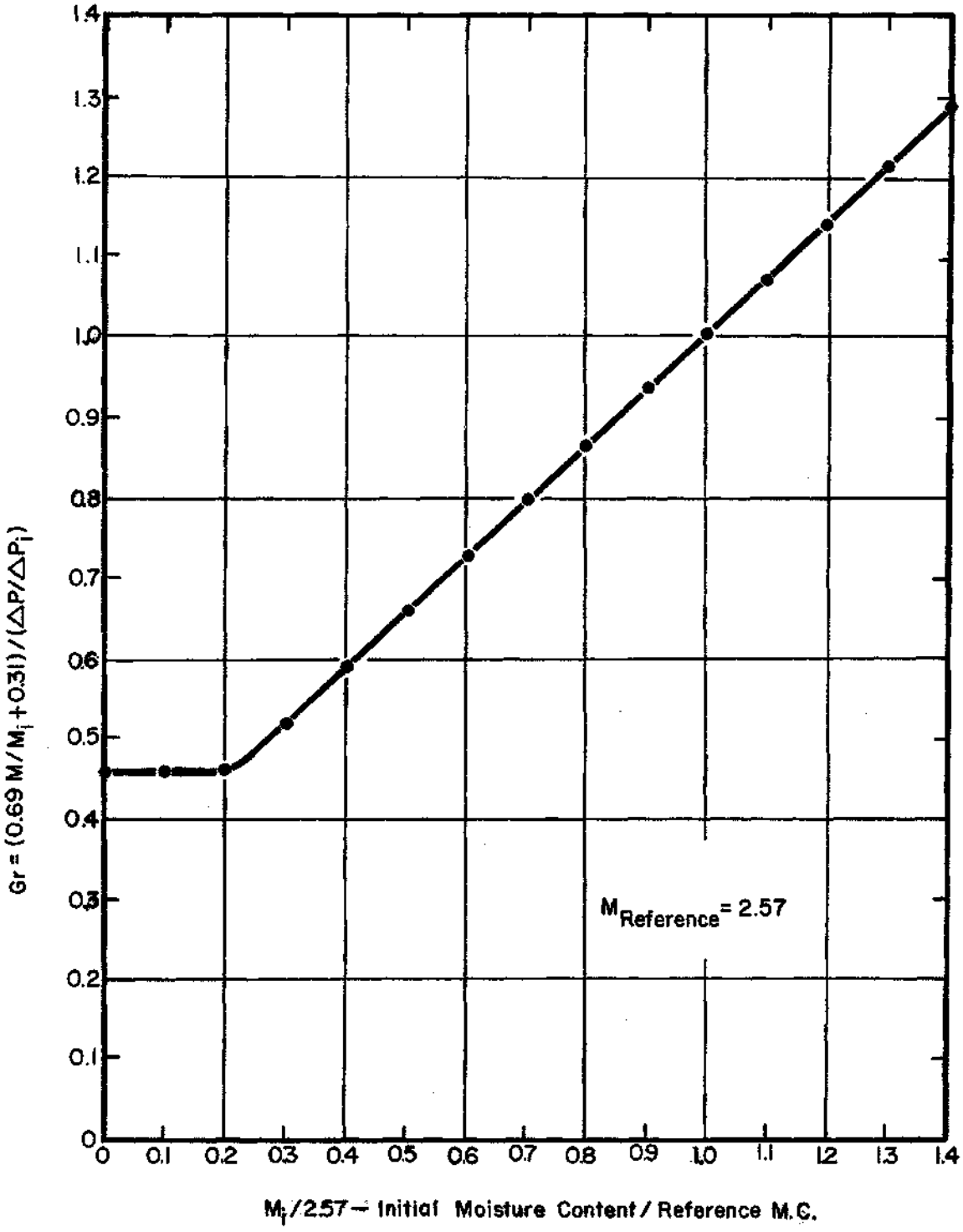


Figure 11. Gr vs Moisture Content Ratio

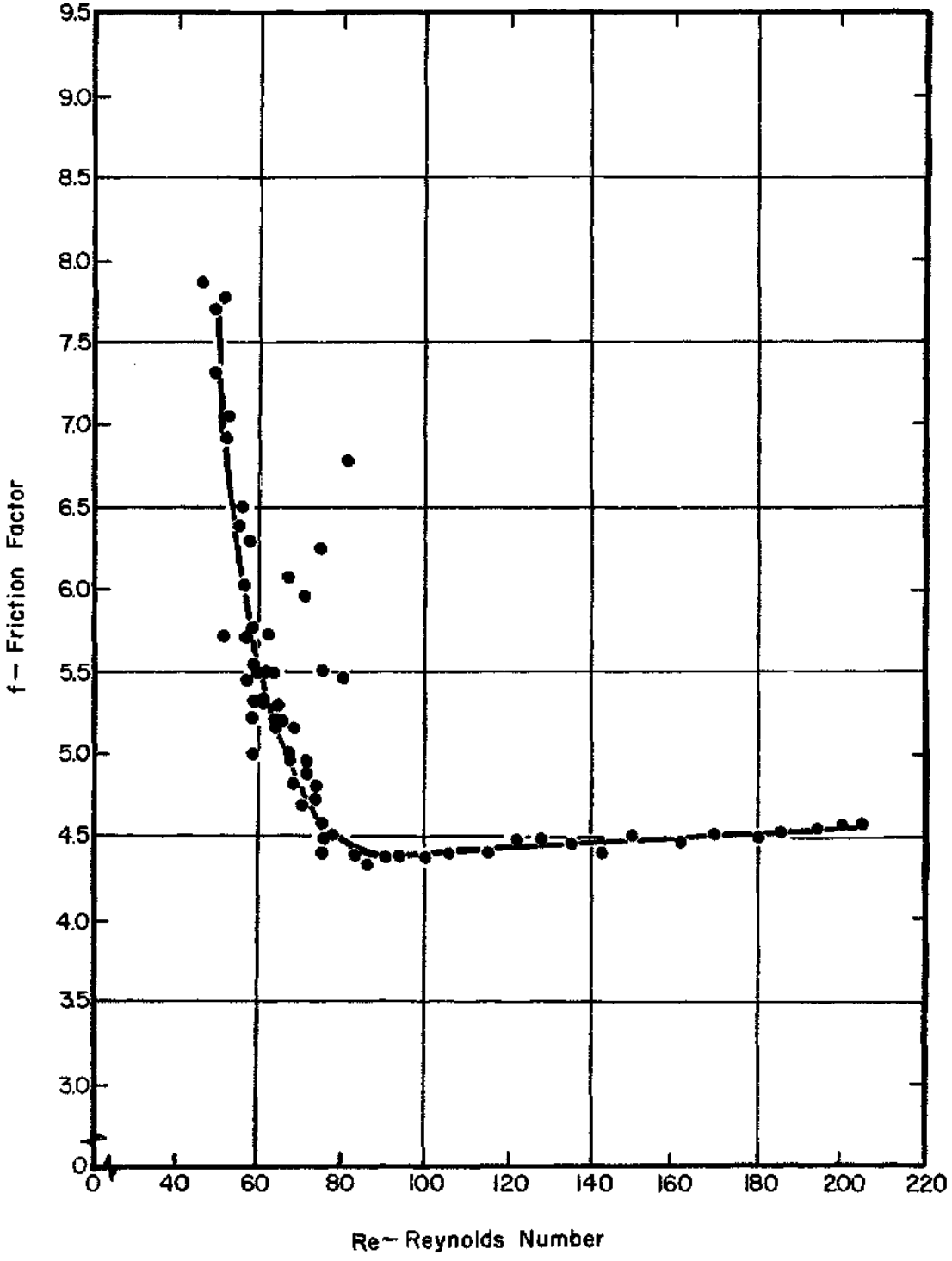


Figure 12. Dimensionless Friction Factor vs Reynolds Number

variables well. The weakness of the correlation is realized but it appears to be the best method available when dealing with a random shape material as was used in this investigation.

Rate of Drying Curves

The rate of drying curves, Figures 14 and 29, were obtained by determining the slopes of the drying curves with a Gerber Derivimeter, Model D-2 slope reader. The accuracy of the rate of drying curves is therefore not as high as that of the original drying curves. A number of the curves are included for completeness and to give an indication of the shape of the rate of drying curves.

Effect of Bed Thickness

The variation in drying time with bed thickness is shown graphically in Figure 13, along with the corresponding rate of drying curves in Figure 14. Figures 25, 26, 27, and 28, in Appendix B show similar results. It is noted that the drying time decreases with decreases in bed thickness.

Effect of Temperature

The variation in drying time with temperature is shown in Figures 15a and 15b. The temperature was varied from room temperature up to 335°F. The rate of drying increased with temperature; however, the heat required to raise the air to the drying temperature also increases.

Effect of Flow Rate

The variation in drying time with flow rate is shown in Figure 16, page 53. Similar curves shown in Figure 31 in Appendix B, page 110. The

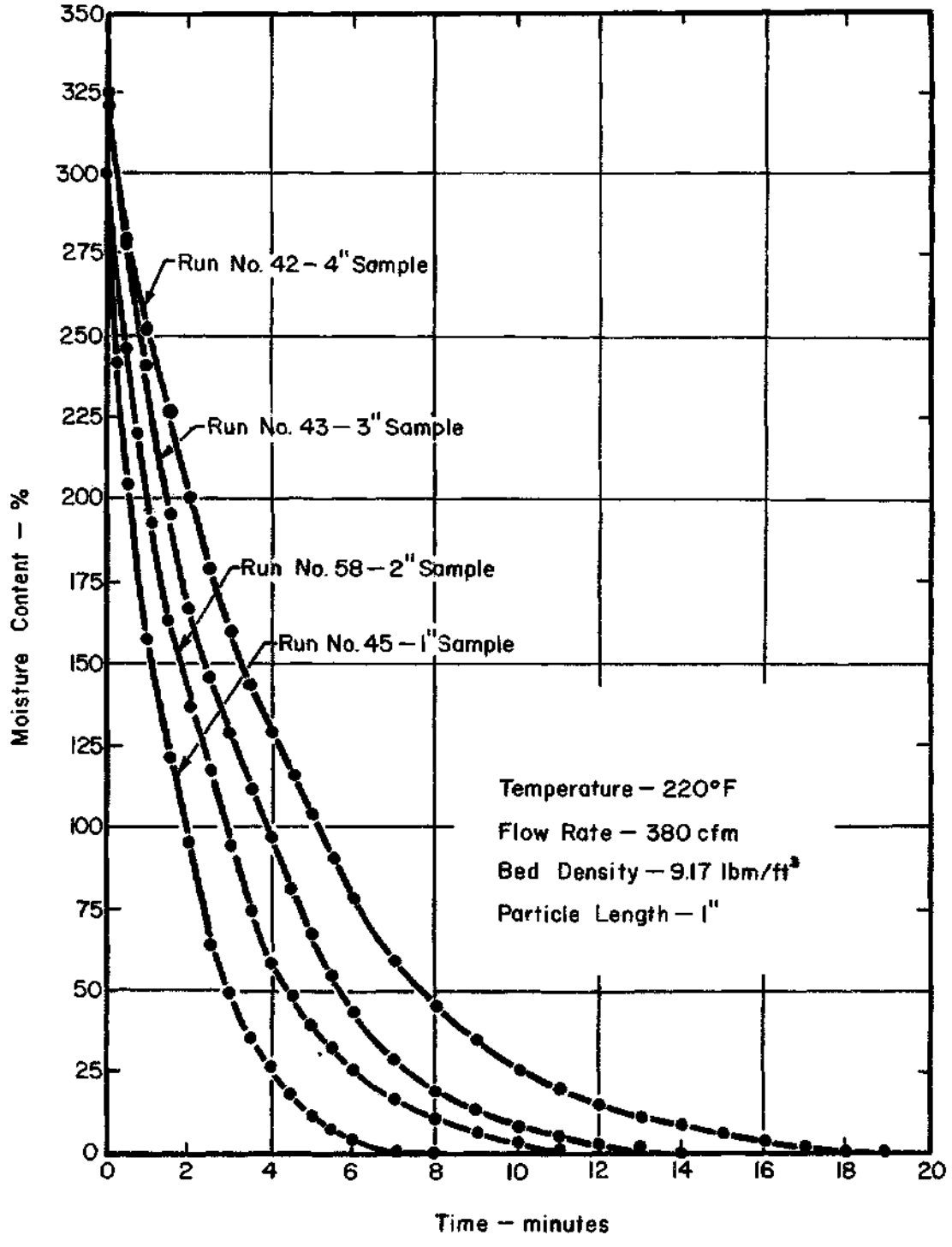


Figure 13. Drying Curves for Various Bed Thicknesses

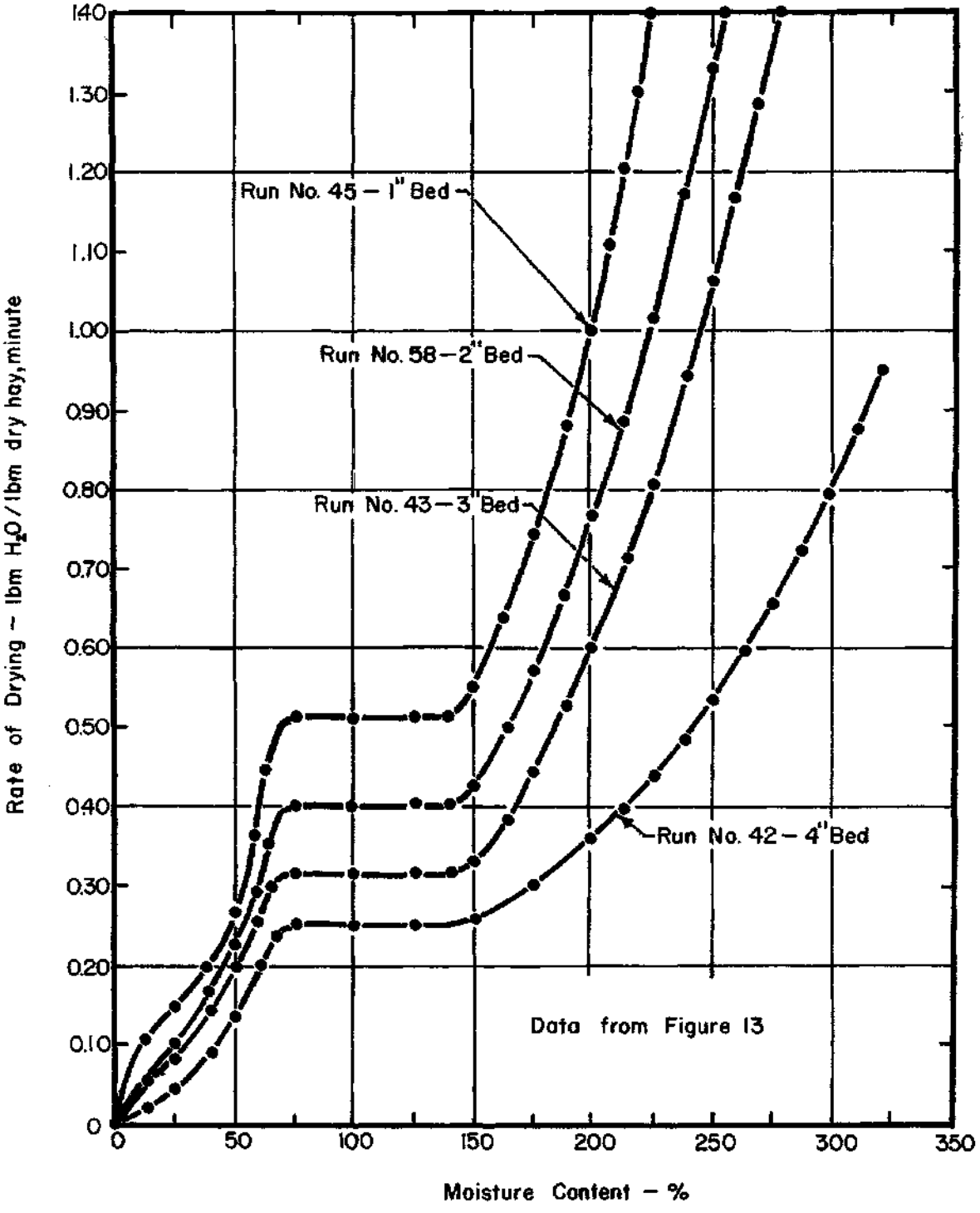


Figure 14. Rate of Drying Curves for Various Bed Thicknesses

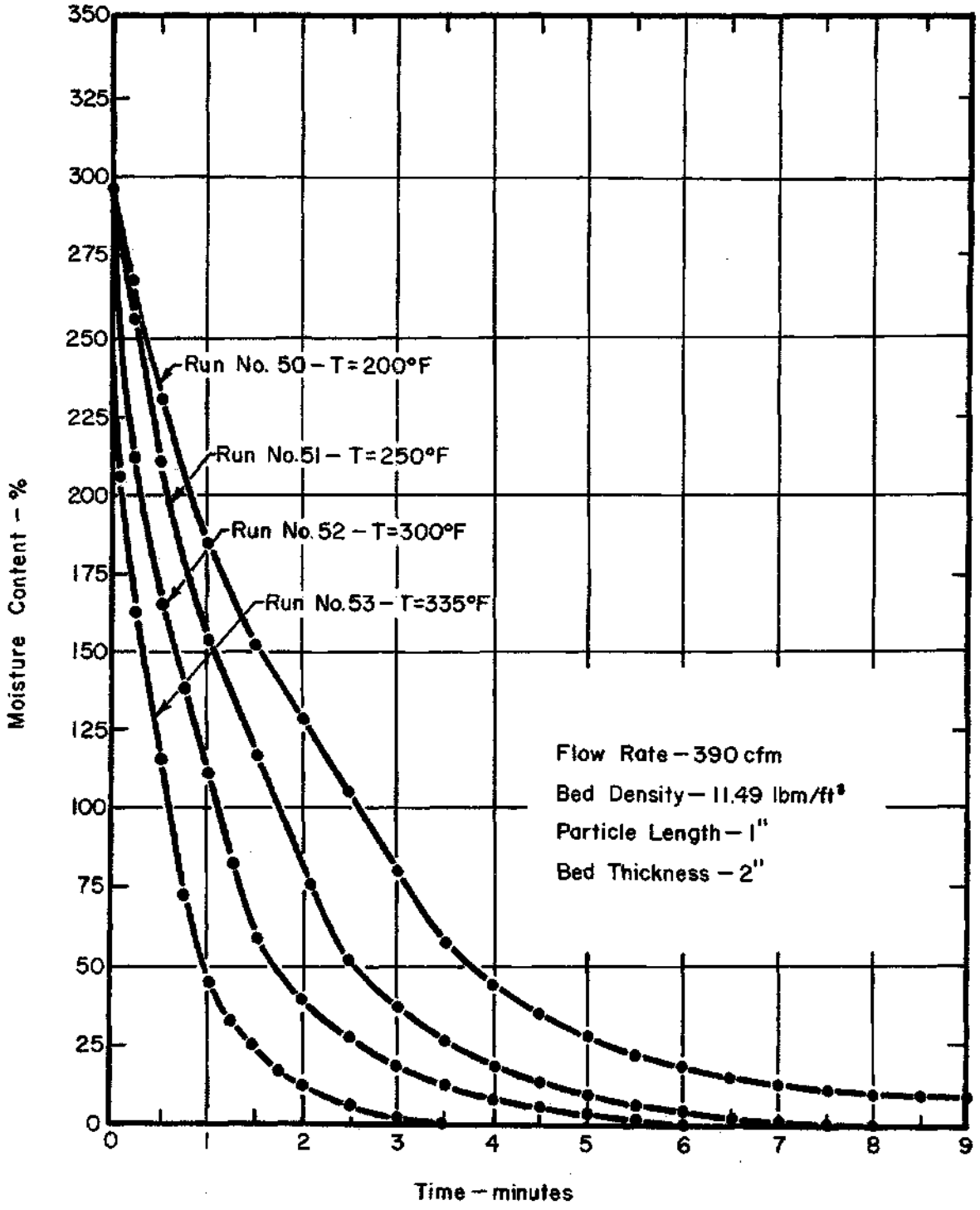


Figure 15a. Drying Curves for Various Temperatures

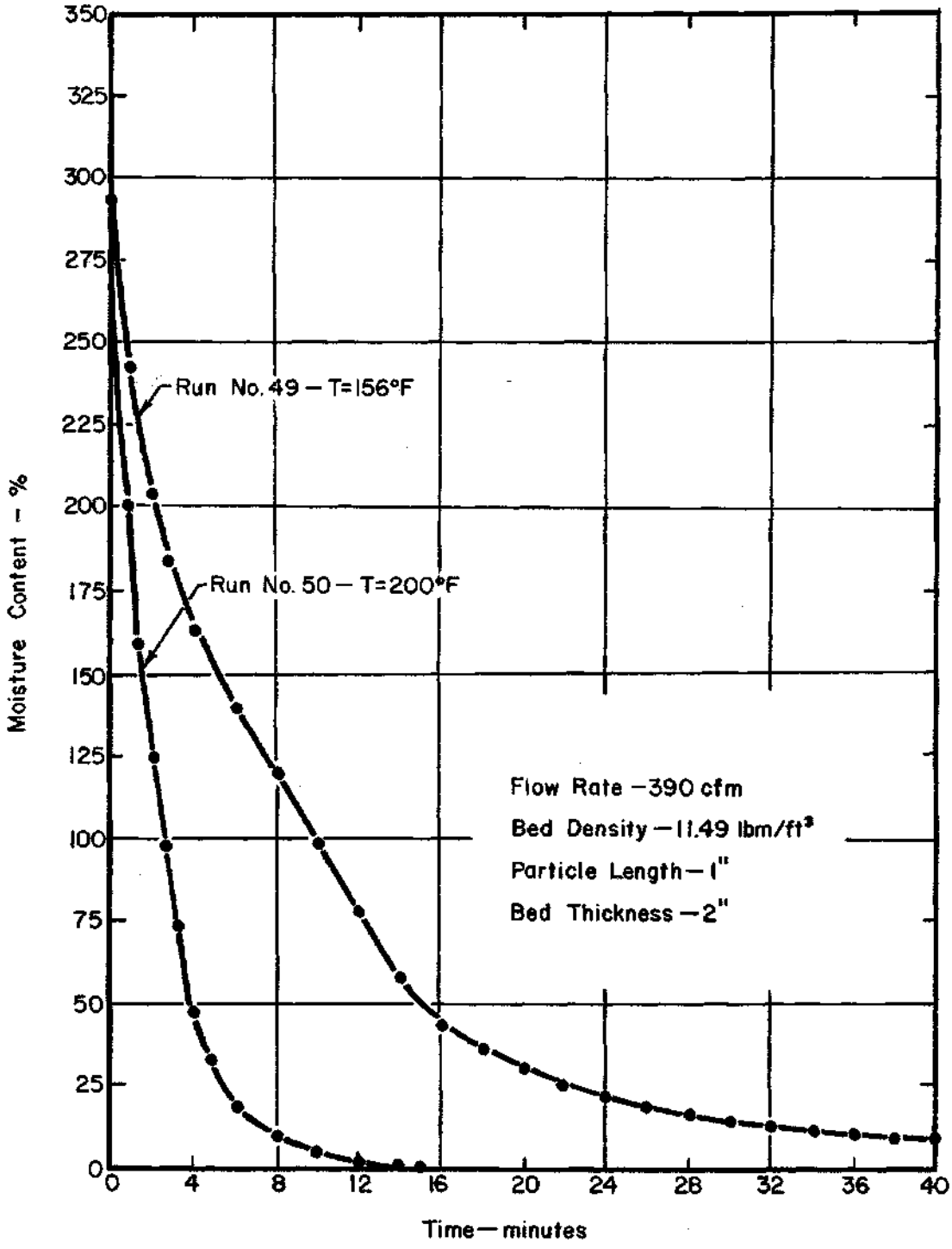


Figure 15b. Drying Curves for Various Temperatures

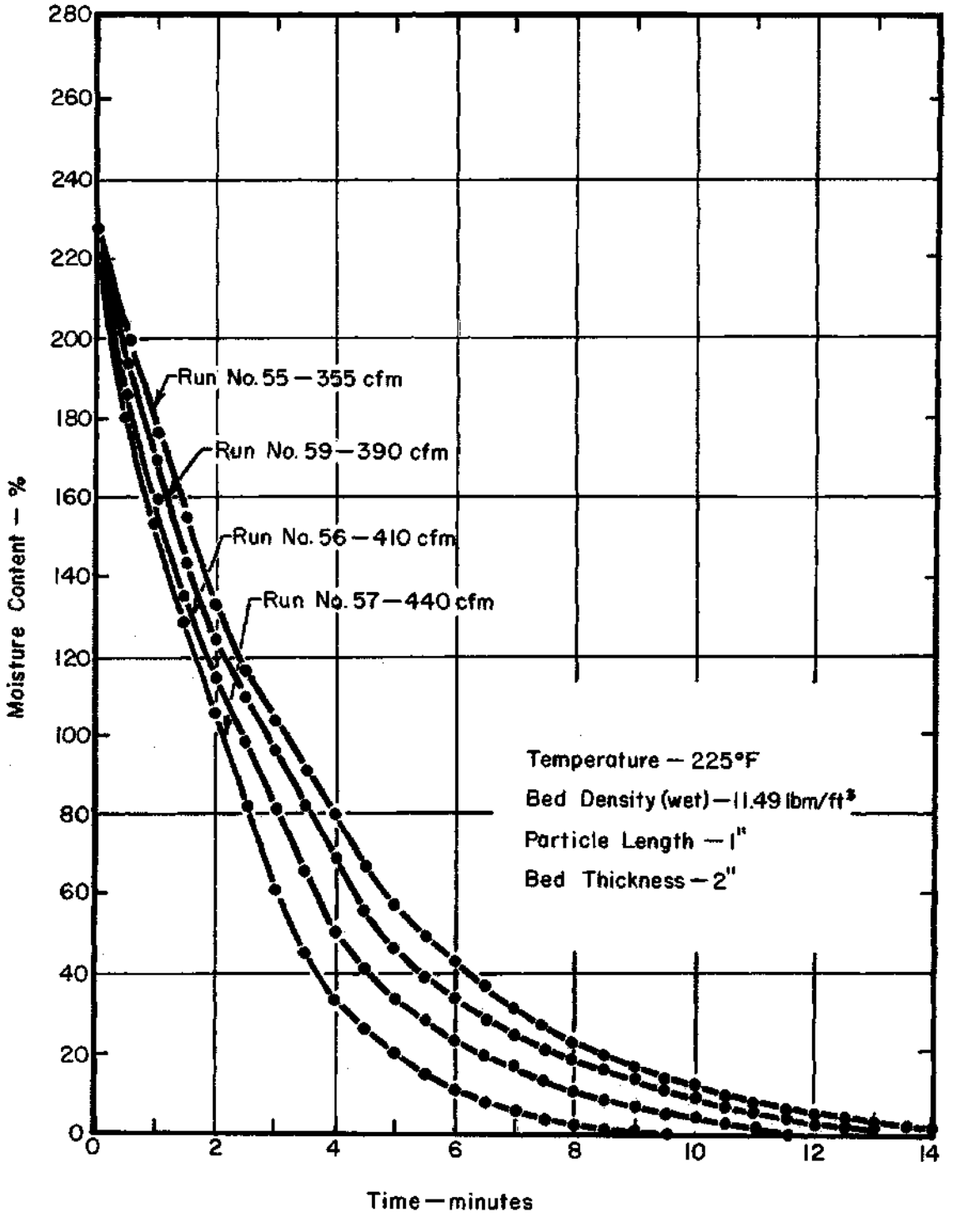


Figure 16. Drying Curves for Various Flow Rates Through Bed

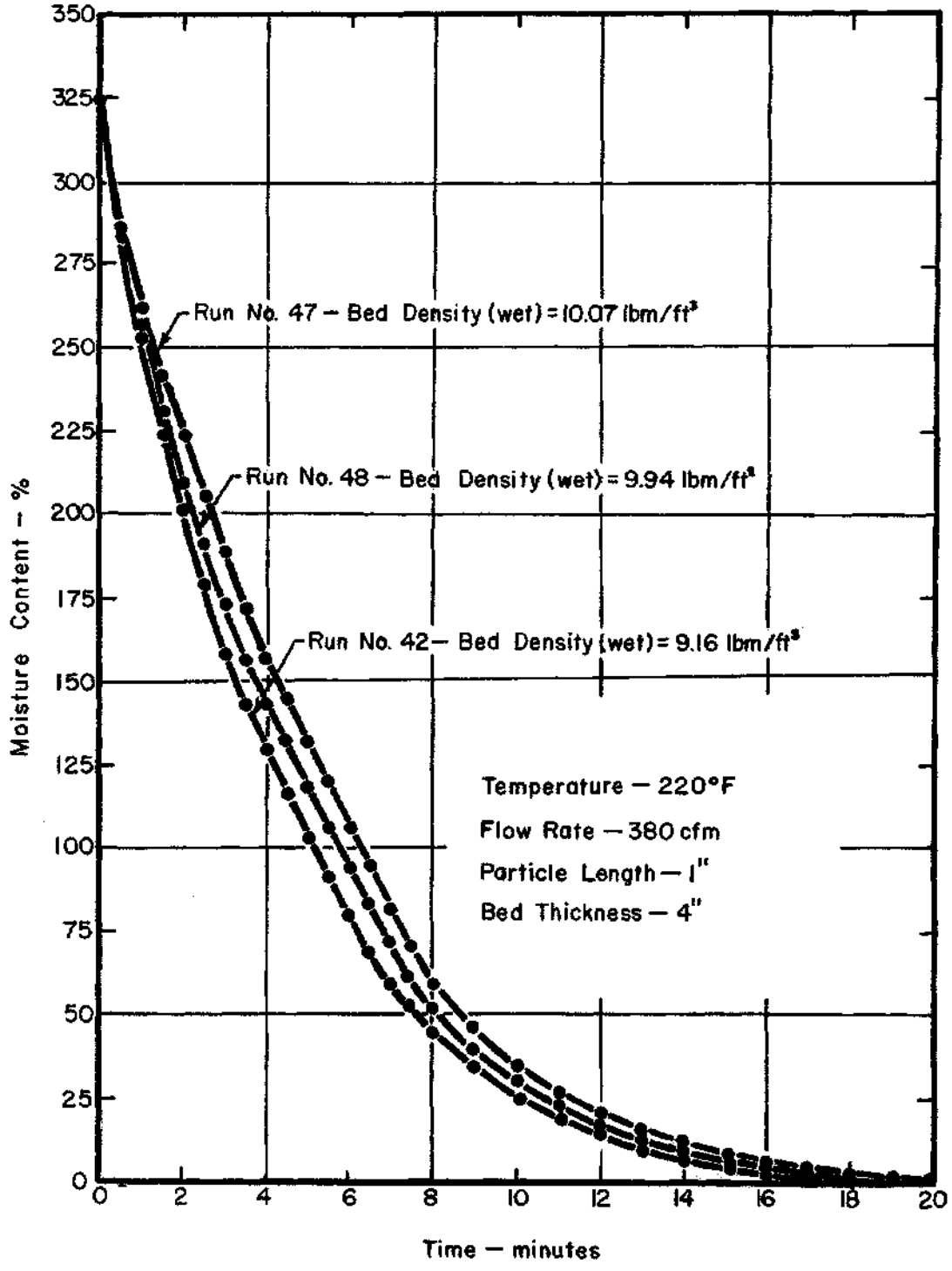


Figure 17. Drying Curves for Various Bed Densities

flow rate was varied from 300 to 500 cfm for the drying runs and over a range of from 300 to 910 cfm for the pressure drop measurements. The drying time is seen to increase with decreasing flow rate. The rate of drying during the constant rate period was found to vary as the square of the velocity of the air flowing through the bed indicating the flow to be turbulent.

Effect of Bed Density

The variation in drying time with bed density is shown in Figure 17, and in Figures 32 and 33 in Appendix B. The bed density (wet) was varied between 9.16 and 13.75 lbm/ft^3. The drying time decreases with decreasing packing density. The wet density appears to be a better variable for correlation than the dry density since it is related to the volume of the bed during the initial periods of drying when the drying rates are greatest. Also the material will always be wet when packed. Although the beds were completely filled with material in each run, the volume actually occupied by the solid material was small when compared to the total volume of the bed. The porosity of the bed thus varied from about 80 per cent when wet to approximately 95 per cent when dry.

Effect of Length of Material

The variation in drying time with particle length is shown in Figure 18. The drying time decreases with decreasing length of material. The one-half inch material dried 33 per cent faster than the uncut material. This indicates that some degree of chopping is necessary. The one inch material dried 28 per cent faster than the uncut material. This is 5 per cent less than the one-half inch material and 10 per cent

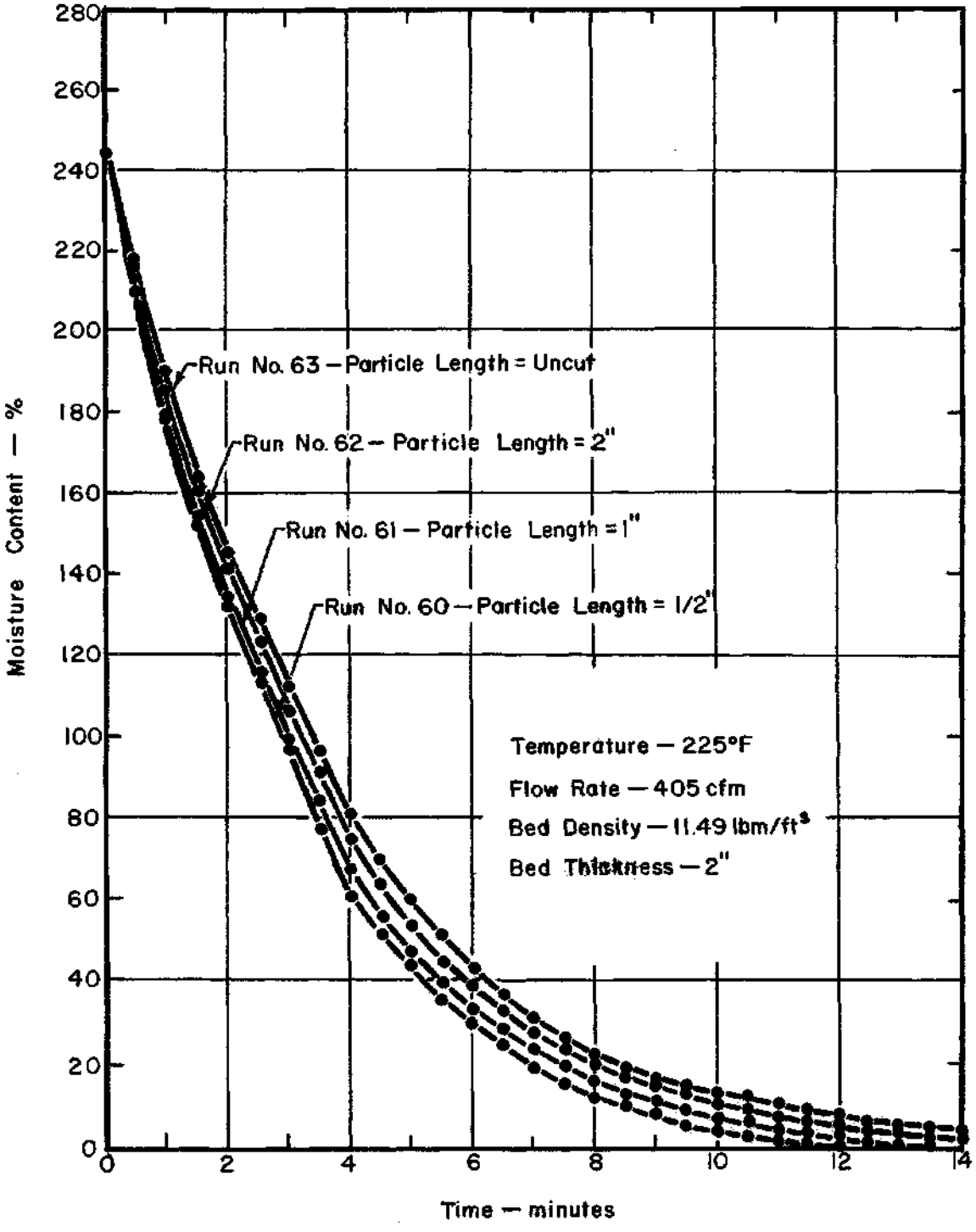


Figure 18. Drying Curves for Various Particle Lengths

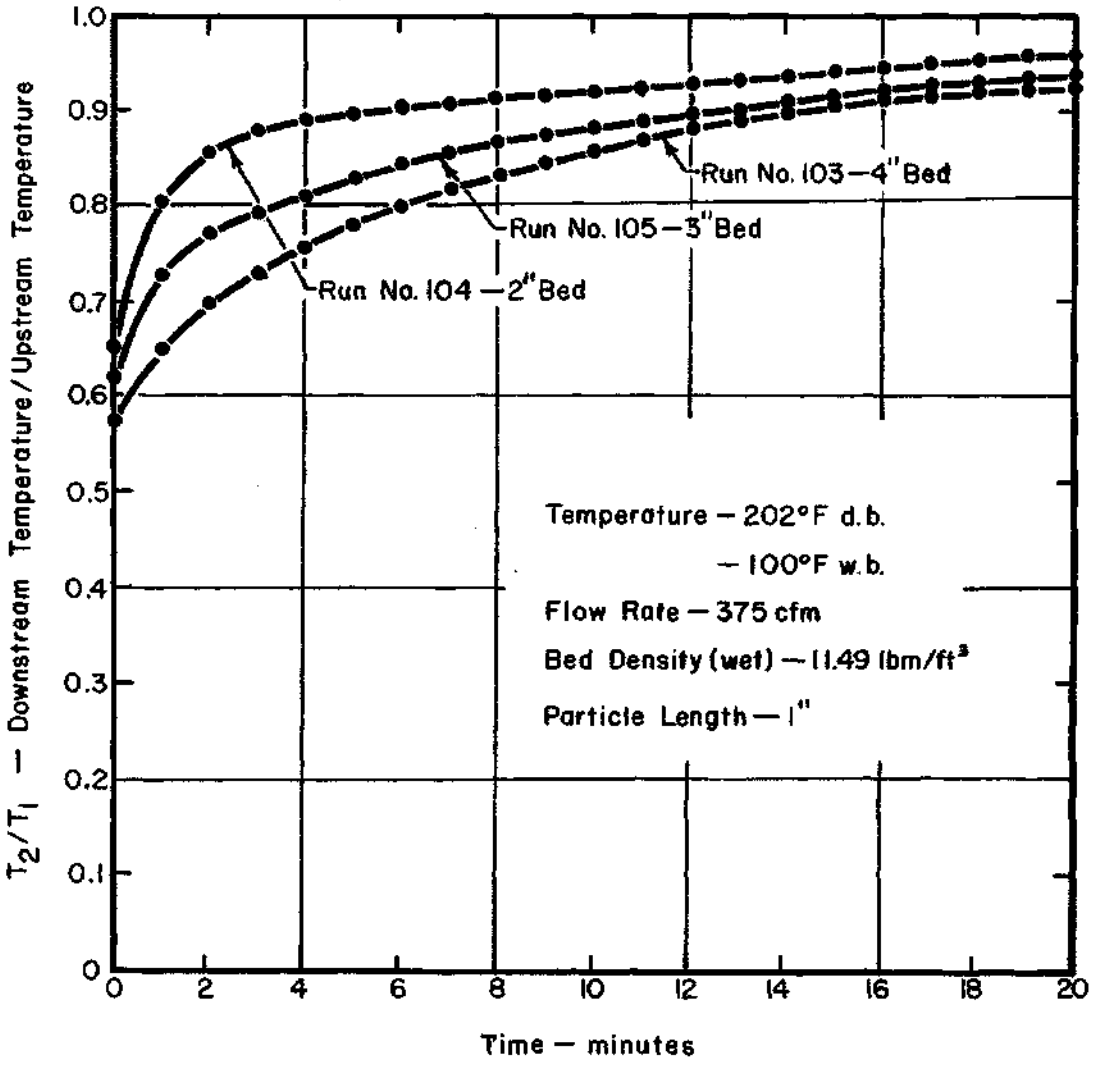


Figure 19. Variation in Downstream Dry Bulb Temperature During Drying

greater than the two inch material indicating that the observed increase in drying rate becomes less as the particle length is decreased.

Initial Moisture Content

The initial moisture content depends on the stage of growth of the material, the time of day when the material is cut, and on the existing weather conditions; therefore, this variable could not be controlled. The drying rates in all regions of drying will be different due to variations in this variable as is observed in Figures 17 and 33. Due to factors mentioned above, it was impossible to hold the temperature, flow rate, bed thickness, bed density, and particle length constant; and then vary the initial moisture content. During this investigation the initial moisture content (dry basis) varied between 119 and 447 per cent.

Downstream Wet and Dry Bulb Temperatures

The variation in the wet and dry bulb temperatures of the free stream air during drying is shown in Figures 19 and 20. The corresponding drying curves are shown in Figure 27. The largest variations occurred during the initial adjustment and constant rate periods of drying with the variation slowly decreasing as drying continued into the falling rate period.

Variable Flow Rate

In a commercial through-flow conveyor type drier the velocity during drying will not be constant except at a given point in the drier since the moisture content will vary as the material moves through the drier, thus causing the flow rate to vary. Therefore, for completeness,

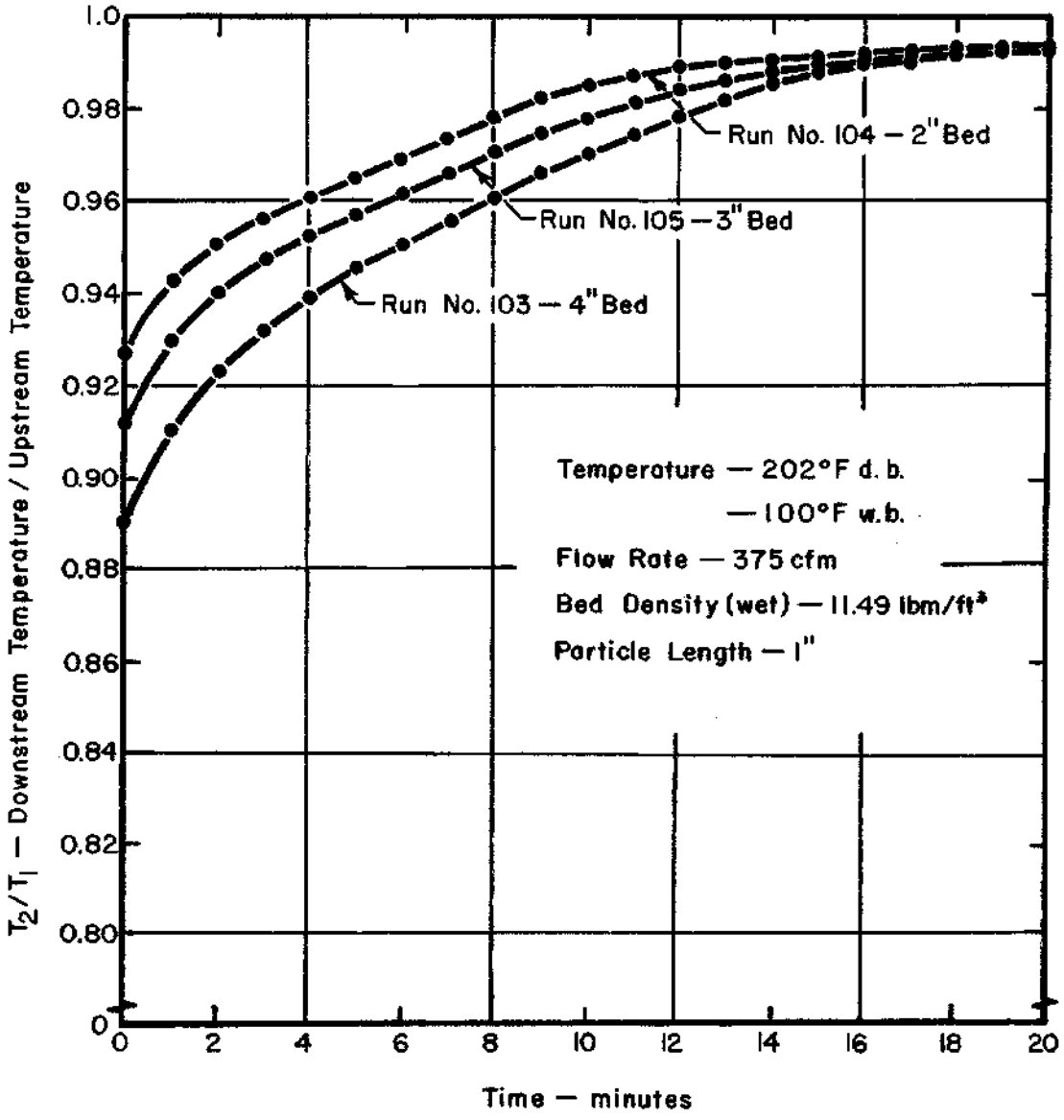


Figure 20. Variation in Downstream Wet Bulb Temperature During Drying

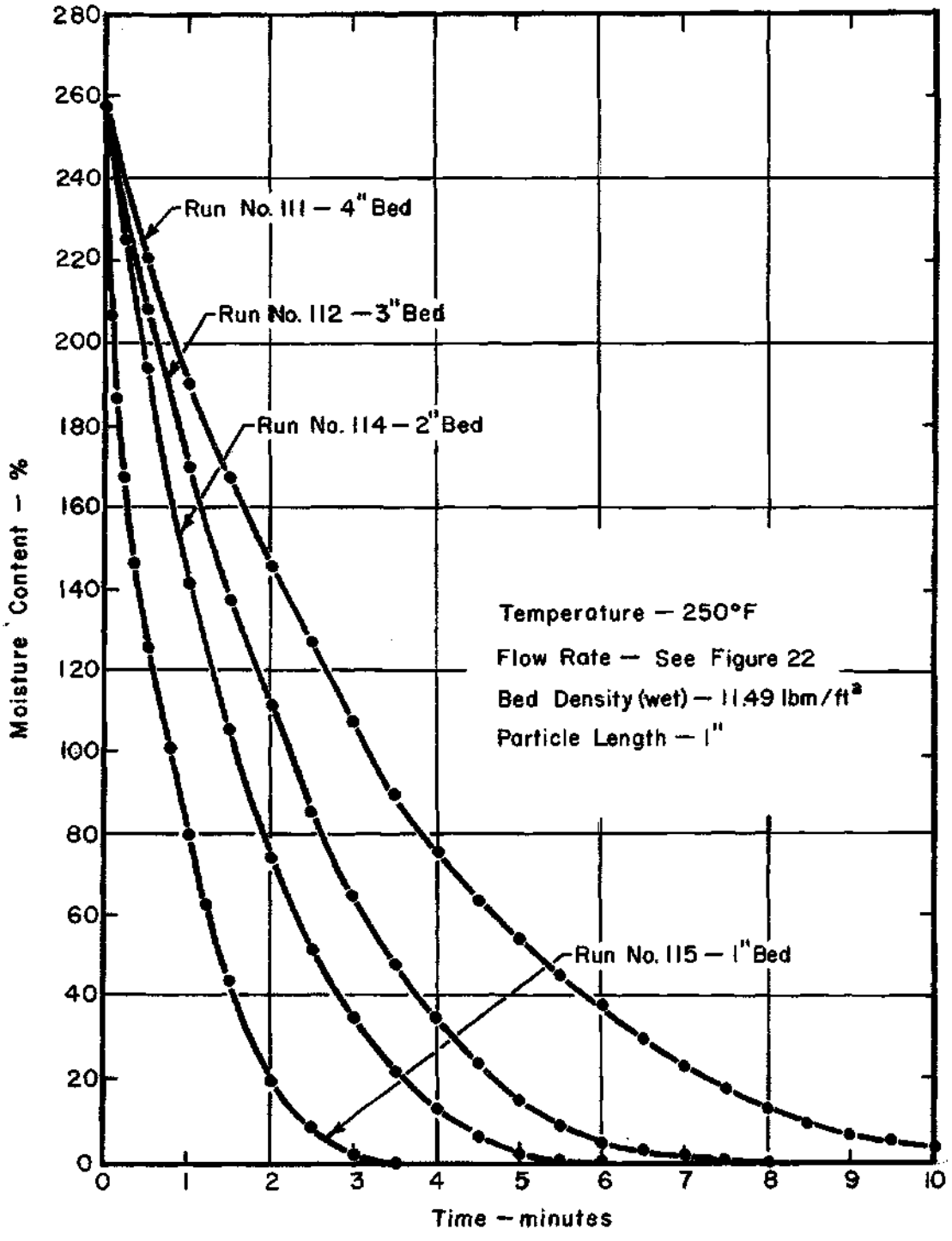


Figure 21. Drying Curves for Various Bed Thicknesses with Varying Velocity

a series of runs was made allowing the velocity to vary during the run. The results of these runs are shown in Figures 21 and 22. By comparing Figures 21 and 28, it can be seen for the range of conditions tested, that allowing the velocity to vary is equivalent to increasing the temperature approximately 20°F and holding the through-flow velocity constant. As occurred with the pressure drop under a constant flow rate the most significant variation occurred during the first period of drying when the variation decreased as drying proceded into the falling rate period; however, the variation was only about 10 per cent of that occurring during the constant flow rate runs. Due to the variation in velocity, a constant rate period was not observed. Other than this, the curves did not differ appreciatively from those with a constant flow rate.

Physical Properties

Very little physical property data is available on hay and no property data was found on coastal bermuda grass. For this reason data on other agricultural products which were similar to coastal bermuda grass was used in the calculations. A value of 0.25 BTU/lbm °R was used for the specific heat of dry hay and a value of 65.0 lbm/ft^3 was used for the density of solid hay.

Critical Moisture Content

For the solution of the equations developed in this investigation, it was necessary to know values of the critical moisture content. Such values can only be obtained by actually running tests.

Chemical Aspects

Although this investigation was not directly concerned with the

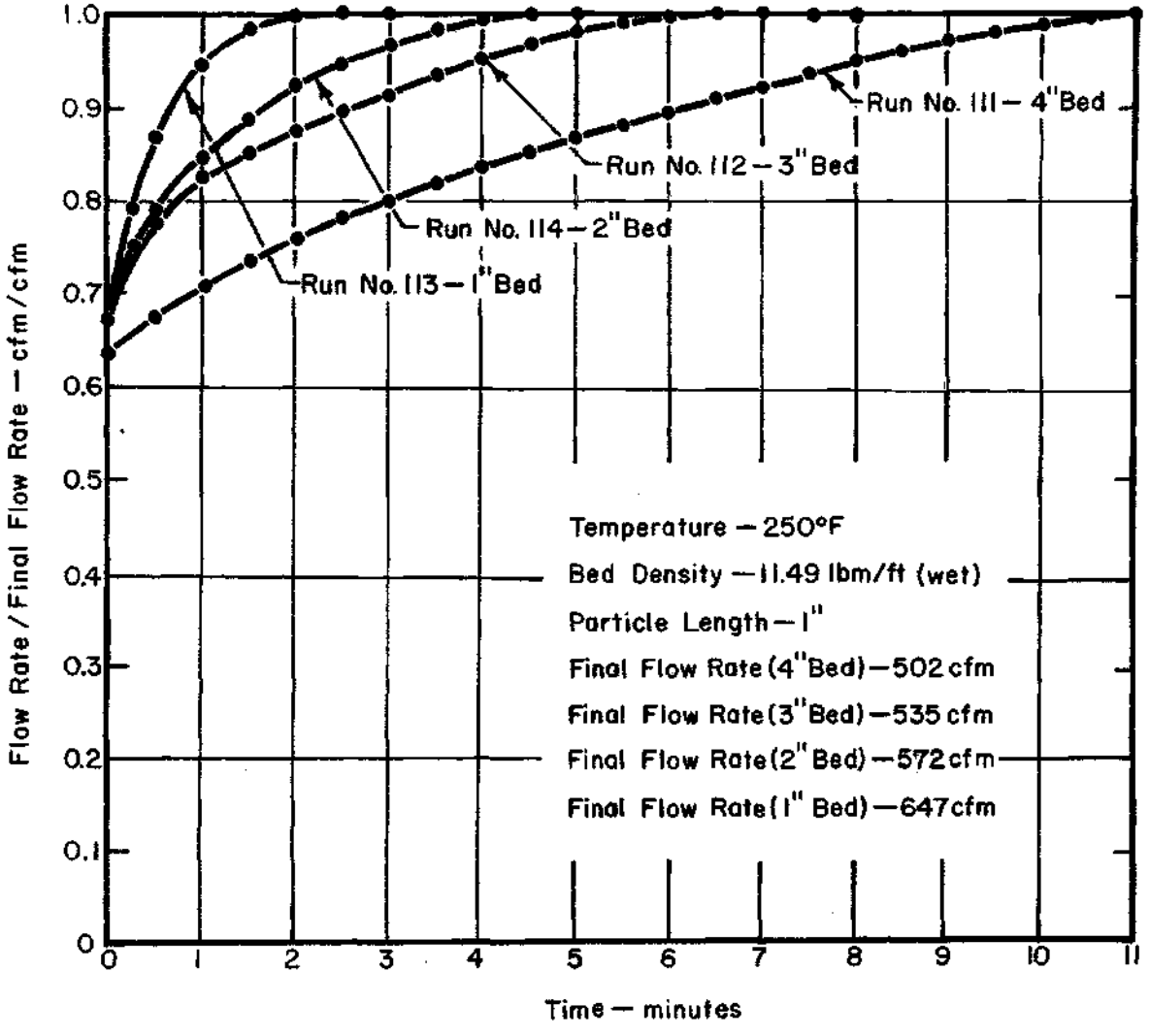


Figure 22. Variation in Flow Rate Through Bed During Drying

chemical aspects of hay drying, it is felt worthwhile to mention the observed effects of drying on the food value of the material. Analysis of the composition of various samples were made by the Georgia Department of Agriculture and are shown in Table 2, Appendix C. Run No. 101 is a field dried sample. The drying time was approximately two days. Run No. 74 is a sample of the same material artificially dried in the test drier. Comparing the crude protein in runs number 74 and 101, it is observed that the same material when dried in the sun has only half the food value as that of the artificially dried material.

CHAPTER VII

COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

The equations derived in Chapter III were solved in general form by numerical methods on the Burrough's 5000 Digital computer for the initial adjustment and constant rate period. A detailed description of the method of solution is given in Appendix A. The experimental result relating the linear variation in moisture content with pressure drop across the bed was used in an analogy to determine the variation in the local mass transfer coefficient during drying. Very small time intervals had to be taken to insure stability of the difference equations. A detailed analysis to determine the interval of stability was difficult. By making certain simplifications, it was found that the distance interval depended on the ratio of the molar flow rate to the local mass transfer coefficient times the interfacial area. The time interval required for stability was so small that the time required to solve the equations prohibited their usage. Ironically the time required to solve the equations was approximately five times the time required to dry the sample experimentally. The program for the computer is given in Appendix D. Figure 23 shows a drying curve determined for the general equations along with the experimental curve. Figure 24 shows the bed temperature distribution at various times as determined from the solution of the general equations.

Due to the time (approximately one hour) required to solve the general equations, the equations were simplified by removing terms which

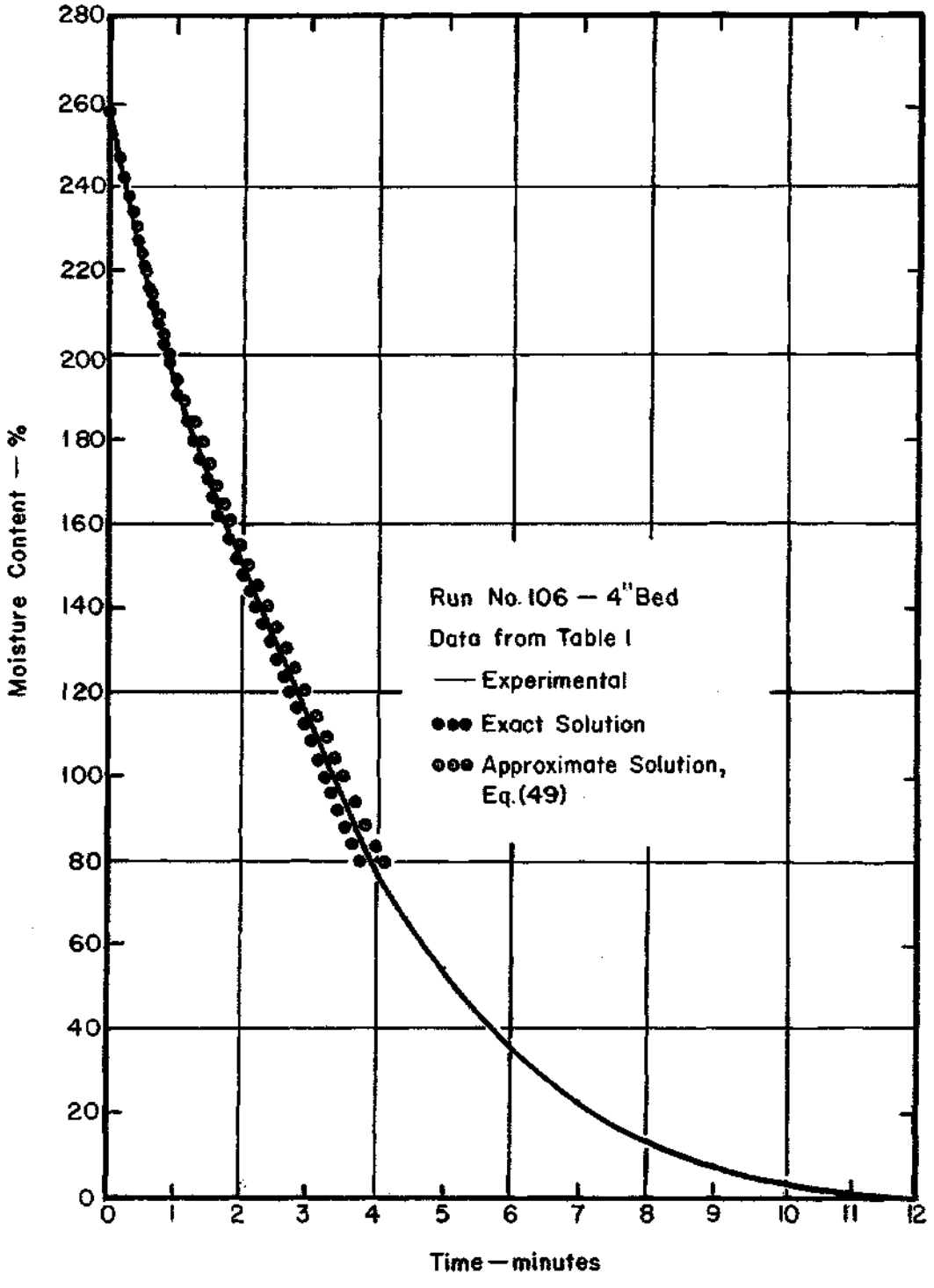


Figure 23. Comparison of Experimental and Analytical Results

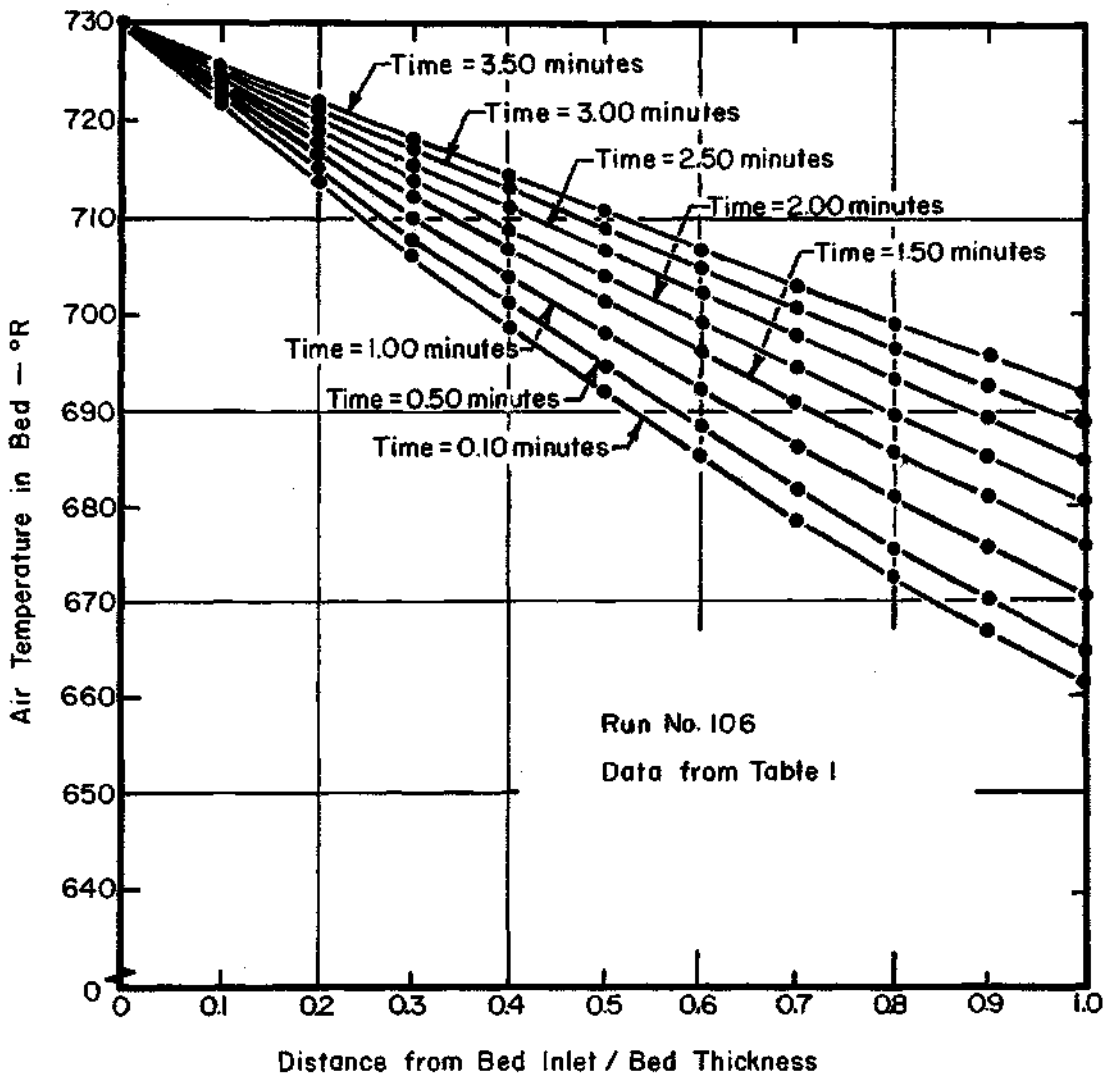


Figure 24. Temperature Distribution Through Bed During Drying as Predicted from Numerical Solutions

were small when compared with other terms in the equations. The simplified equations are shown in Appendix A. Again by using the analogy between momentum and mass transfer along with the experimentally determined relationship between the pressure drop and moisture content, the local mass transfer coefficient was determined. From the general analytical solution it was observed that the variation in the drying rate occurring during the initial adjustment period was due more to the change in the mass transfer coefficient than to the adjustment in the surface temperature. When the mass transfer coefficient was considered as constant in the general equations, the shape of the rate of drying curve was similar to the experimental curve except for the sharp decrease in the rate of drying curve occurring during the initial adjustment period in the experimental curves. Thus in the simplified equations the surface temperature was assumed to be constant and equal to the wet bulb temperature of the free stream mixture. The reduced equations were then expanded into a series and solved resulting in Eq. (49a) and (49b). The drying curve determined from this equation is shown in Figure 23, along with the experimental curve and the curve determined from the general equations. As can be seen, the error inflicted by neglecting the terms in the general equation was small. Table 4, page 127, shows a comparison between the experimental drying time, between the initial and critical moisture content, and the drying time determined from Eq. (49). The maximum deviation was 35 per cent.

Due to the complexity of the falling rate period, an approximate method was used to determine the drying rates. This analysis is given on page 88 of Appendix A. By combining the resulting equation with Eq. (49), page 85, the total drying time can be determined. The resulting

equations are given on page 90. It should be noticed that the time required for drying in the falling rate period is extremely sensitive to the value used for the final moisture content, e.g., if the final moisture content is one per cent, the drying time is 1.53 times greater than if the final moisture content was five per cent. For this reason a comparison between the total analytical and experimental drying time is not given. However, in most commercial applications, it is desired to dry to a certain moisture content. For such applications Eqs. (55a) and (55b), page 90 would be very useful. A typical calculation is shown in Appendix A, page 99.

It should be noted that the constants of Eq. (55), page 90 are actually groups of variables. It can be seen that there are seven independent variables along with numerous dependent variables. The independent variables are air temperature, concentration of water vapor in the free stream, flow rate, moisture content, bed density, bed thickness, and length of material.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

An analytical and experimental investigation of the through-flow drying of a bed of hay has been made. The analytical study treated the general theory of through-flow drying in a bed while the experimental work consisted of a study of the variables involved. The observed experimental results were then combined with reduced analytical equations to produce equations for calculations. The conclusions from these investigations are:

1. In the range of variables tested, the pressure drop across a bed of hay is approximately a linear function of the moisture content between the initial and critical moisture content. The pressure drop across a wet bed is approximately one-half the pressure drop across a dry bed.

2. The drying time in through-flow drying of hay decreases with decreasing bed thickness, decreasing material length, decreasing packing density, increasing flow rate and increasing temperature.

3. The drying time during the initial adjustment and constant rate periods can be analytically predicted within 35 per cent.

The following are recommended as logical extensions of the work which has been presented:

1. A wider range of tests should be performed in order to determine the optimum bed thickness. The equations obtained in the present investigation could be used to analytically check the experimental results.

2. The same investigation presented here should be performed using different material such as corn silage, alfalfa, etc.

3. Both an analytical and experimental investigation of the through-flow drying of a bed of agricultural products should be performed allowing the velocity to vary during drying, i.e., consider a constant pressure drop across the bed during drying.

4. A more extensive study of the effect of drying methods on the food value of the dried product should be performed. This would require analysis of each sample similar to those shown in Table 2, Appendix C, page 124.

APPENDIX A

ANALYSIS

The design of a commercial drier requires a knowledge of two quite distinct phenomena - the rate of drying under any given conditions and the rate of air flow through the bed of material. The analytical prediction of these rates is complicated by the transient conditions occurring in the bed. In drying agricultural products, from 50 to 80 per cent of the initial mass of the bed is removed during drying. Corresponding to this mass loss is a decrease in porosity which causes the pressure drop through the bed to decrease substantially for a constant flow rate. This also affects the heat and mass transfer coefficients, further complicating the problem. The theory proposed in Chapter III of this investigation will be combined with experimental results to give usable equations in the following three Sections: (i) the Resistance to Air Flow - A Correlation of Results, (ii) The Rate of Drying During the Initial Adjustment and Constant Rate Period, and (iii) the Rate of Drying During the Falling Rate Period.

The Resistance to Air Flow - A Correlation of Results

The resistance offered by a bed of material to air flow can be approached from two extremes, either by considering the resistance offered to the air by a single particle and extending this to the case where there are many particles,[15,16,35,36] or the problem can be regarded as one of fluid flow through a pipe in which the fluid path is altered by a presence

of solid particles. These concepts cause a pressure drop either by expansion and contraction losses when a fluid flows through a series of interconnecting channels varying in shape, cross-section, and direction,[37] or by the transfer of momentum which occurs between air and the front and back faces of the particles comprising the bed.[36]

In this investigation the concept of flow around particles will be used since more is known about the number and size of particles than is known about the complex flow channels. Since the particle size varies as the material dried a correlation between a dimensionless friction factor and modified Reynolds number will be made only for dry material. A procedure for predicting the pressure drop in the wet bed using the data for the dry bed will then be derived.

The Pressure Drop Through a Dry Bed

The modified Reynolds number and friction factor presented by Brownell and Katz[38,39] will be used to correlate the pressure drop across a dry sample. The modified Reynolds number is defined as:

$$Re = \frac{D_p G}{\mu (P_o) A_o} \qquad \ldots\ldots (26)$$

Where: A_o = cross sectional area of the bed, ft^2

D_p = particle diameter, ft

G = mass flow rate per square foot of bed, $lbm/min\ ft^2$

Re = Reynolds number

μ = viscosity of air, lbm/ft, sec

The friction factor is defined as:

$$f = \frac{2g_c D_p \Delta P_d (P_o) \rho A_o^2}{LG^2} \qquad \text{.... (27)}$$

Where: f = friction factor

g_c = constant - 32.2 lbm ft/lbf - sec^2

L = depth of bed, ft

ΔP_d = pressure drop across the dry bed, lbf/ft^2

ρ = density of air, lbm/ft^3

The porosity of the bed is obtained by assuming that the material in the bed is solid. With this assumption we can write:

$$V_v = V - V_m$$

Or:

$$V_v = V - \frac{m_h}{\rho_h} - \frac{m_w}{\rho_w} = V - m_h\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)$$

Rearranging:

$$P_o = \frac{V_v}{V} = 1 - \frac{m_h}{V}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right) \qquad \text{.... (28)}$$

Where: m_h = mass of dry material, lbm

m_w = mass of moisture contained in bed, lbm

M = moisture content, lbm moisture/lbm dry material

V = total volume of bed, ft^3

V_m = volume occupied by the material, ft^3

V_v = volume of voids in bed, ft^3

ρ_h = density of hay, lbm/ft^3

ρ_w = density of water, lbm/ft^3

The diameter of the particles is obtained by considering the particles as cylinders. The number and length of the particles are known, hence the diameter can be obtained by:

$$D_p = \left[\frac{4 \ell m_h}{\pi n}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right]^{1/2} \quad \ldots\ldots (29)$$

Where: ℓ = length of particle, ft

n = number of particles

For a bed of dry particles as will be used in the correlation Eq. (29), reduces to:

$$D_p = \left[\frac{4 m_h}{\pi \ell n \rho_h}\right]^{1/2} \quad \ldots\ldots (30)$$

The Pressure Drop Through a Wet Bed

Due to the variation in cross-sectional flow area during drying, the correlation given by Eqs. (26) and (27) will not hold. Therefore, a method based on experimental results for taking the pressure drop through dry beds and extending it to a wet bed will be presented.

The variation in pressure drop through a bed of wet material during drying was recorded and is shown in Figure 10, page 44. When these data are plotted as the pressure drop/initial pressure drop versus the moisture content, it is found that the pressure drop/initial pressure drop is approximately constant up to the critical moisture content and then varies approximately linear with moisture content. Each curve has approximately the same shape and slope. In order to reduce the data to a single curve a reference curve was chosen and the pressure drop/initial

pressure drop taken from the reference curve, but corresponding to the initial moisture content of the curve in question was multiplied by the equation of the curve. The correction factor is given the symbol, Gr. This moisture content is then divided by the initial moisture content. The resulting plot is shown in Figure 11, page 47. The experimental pressure drop data for each run checked within experimental accuracy with the single reference curve. The resulting equation applying between the initial moisture content and critical moisture content is as follows:

$$\frac{\Delta P}{\Delta P_i} = \frac{1}{Gr}\left(0.69\,\frac{M}{M_i} + 0.31\right) \qquad \ldots\ldots (31)$$

Where: ΔP_i = initial pressure drop across the bed, lbf/ft^2

Gr = correction factor

M_i = initial moisture content, lbm moisture/lbm dry material

Inserting the critical moisture content into this equation gives the pressure drop across the bed of dry material, since the pressure drop is assumed to remain constant, between the critical and final moisture content.

The Rate of Drying During the Initial Adjustment and Constant Rate Periods

The equations derived in Chapter III will be combined and integrated in the following sections to give the rates of drying during the initial adjustment and constant rate periods of drying. A simplified approach will be taken to approximate the falling rate period and thus result in a combined equation for the total drying time.

Relationship Between Concentration and Temperature at the Surface of the Drying Material. During the initial adjustment and constant rate periods the water vapor at the drying surface is saturated and thus knowledge of the temperature is sufficient to determine the vapor pressure. The vapor pressure of water over the entire liquid range from 0°C to the critical temperature (374.15 °C) can be represented by an approximation polynomial equation developed by Bridgeman and Aldrich.[40]

$$\log_{10} P_{AS} = A + Y_2 - B(L + CT_3)Y_3 \qquad \ldots\ldots(32)$$

Where: P_{AS} = vapor pressure at the drying surface, atm

$Y_2 = D(T - 187)/(T + E)$

$Y_3 = 0.01\left[(3\sqrt{3})/(2 \times 1.87^3)\right](Y_1 - 0.01\alpha)\left[1.87^2 - (Y_1 - 0.01\alpha)^2\right]$

$Y_1 = 0.01\ (T - 187)$

$\alpha = Y_4^2\,(1.87^2 - Y_4^2)/F(1 + G1 \times T_s)$

$Y_4 = -1.87 + 3.74\left\{H - K \cosh^{-1}\left[L1/(T_s + M1)\right]\right\}$

and where:

A = 1.06423320	G1 = 3.377565×10^{-3}
B = 1.0137921	H = 1.152894
C = 5.83531×10^{-4}	K = 0.745794
D = 4.16385282	L1 = 654.2906
E = 237.098157	M1 = 266.778
F = 0.30231574	

Solution of General Equations. The solution of the following system of equations along with Eq. (32), page 78 will enable the determination

of the drying rates during the initial adjustment and constant rate periods.

$$\frac{\partial T_s}{\partial \tau} = \frac{(k_{x,loc}a)V}{(m_h c_{ph} + m_w c_{ph})} \left\{ \left[x_A(\tilde{C}_{pA} - \tilde{C}_{pB}) + \tilde{C}_{pB} \right] \left[\frac{Sc}{Pr} \right]^{2/3} (T - T_s) - \frac{x_{AS} - x_A}{1 - x_{AS}} H_{fg} \right\} \quad \ldots\ldots(25a)$$

$$\frac{\partial M}{\partial \tau} = (k_{x,loc}a) \frac{M_A V}{m_h} \frac{x_{AS} - x_A}{1 - x_{AS}} \quad \ldots\ldots(20a)$$

$$\frac{\partial T}{\partial \tau} = \frac{(k_{x,loc}a)(RT)(T_s - T)}{P(P_o)\left[x_A(\tilde{C}_{pA} - \tilde{C}_{pB}) + \tilde{C}_{pB} \right]} \left\{ \left[x_A(\tilde{C}_{pA} - \tilde{C}_{pB}) + \tilde{C}_{pB} \right] \left[\frac{Sc}{Pr} \right]^{2/3} + \tilde{C}_{pA} \frac{x_{AS} - x_A}{1 - x_A} \right\} - \frac{N_B}{1 - x_A} \frac{RT}{P(P_o)} \frac{\partial T}{\partial Z} \quad \ldots(17a)$$

$$\frac{\partial x_A}{\partial \tau} = (k_{x,loc}a) \frac{RT}{P(P_o)} \frac{x_{AS} - x_A}{1 - x_{AS}} - N_B \frac{RT}{P(P_o)} \left[\frac{1}{(1-x_A)^2} \right] \frac{\partial x_A}{\partial Z} + \frac{x_A}{P_o} \frac{\partial P_o}{\partial \tau} + \frac{x_A}{T} \frac{\partial T}{\partial \tau} \quad \ldots\ldots(8a)$$

In Eq. (17a) above, $\partial P/\partial \tau$ is neglected since it is small in comparison to the other term in the equation.

In order to determine the mass transfer coefficient the analogy between momentum and mass transfer is applied. From the analogy, neglecting enterance effects

$$(k_{x,loc}a) = \frac{f}{2} \frac{W_B}{V} (Sc)^{-2/3} \quad \ldots\ldots (33)$$

Substituting Eq. (27) into Eq. (33):

$$(k_{x,loc}a) = \frac{g_c \Delta P\, D_p \rho\, (P_o) A_o^2}{L W_B M_B^2 V} Sc^{-2/3} \qquad \ldots. (34)$$

Where: M_B = Molecular weight of air, lbm/mole

Since both the particle diameter, porosity, and pressure drop vary with moisture content, the quantity $(k_{x,loc}a)$ also must vary. It is assumed that $(k_{x,loc}a)$ is a function of time only at this point.

Substituting Eq. (29) and Eq. (31) into Eq. (34):

$$(k_{x,loc}a) = \frac{g_c \rho\, \Delta P_i\, A_o^2}{V L W_B M_B^2 Gr}\left[\frac{4m_h}{\pi n \ell}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right]^{1/2}\left[0.69\,\frac{M}{M_i} + 0.31\right]\left[1 - \frac{m_h}{V}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right] Sc^{-2/3} \qquad \ldots (35)$$

It should be recalled that Eq. (31) is based on experimental pressure drop data during drying between initial and critical moisture content.

The following are the initial and boundary conditions:

1. At $Z = 0$, $\tau \geq 0$; $T = T_1$, $x_A = x_{A1}$

2. At $\tau = 0$, $Z \geq 0$; $T = T_o$, $x_A = x_{AO}$, $T_s = T_{so}$

By using backwards difference approximations for the partial differentials, the four equations are transformed into difference equations. The difference grid consists of a time and a distance through the bed interval. By selecting a suitable grid size the difference equations can be solved. The numerical procedure was continued until the surface temperature at the outlet from the bed became constant. The

solution to the equations were obtained on a Burroughs 5000 Digital computer. The program is given in Appendix D, page 128 and a comparison with experimental data is given in Figure 24, page 65.

Simplification of General Equation. In the numerical solution of the general equation, the contributions of certain terms were found to be small. By neglecting these terms a closed form solution was obtained.

The following assumptions will be made in the derivations:

1. $\frac{\partial x_A}{\partial \tau} = 0$

2. $\frac{\partial T}{\partial \tau} = 0$

3. $\tilde{C}_{p_{mixture}} \approx \tilde{C}_{pB}$

4. $\frac{\partial P}{\partial \tau} = 0$

5. $x_A \ll 1$

6. $\frac{\partial T_s}{\partial \tau} \approx 0$

7. T_s = constant with distance.

Assumption No. 6 would appear grossly in error at first, however, this assumption was observed to be approximately correct from the general numerical solution. With the above assumption, Eq. (17) reduces to:

$$\frac{\partial T}{\partial Z} = (k_{x,loc}a)(T_s - T)\left[\frac{1-x_A}{W_B}\right]\frac{V}{L}\frac{\left\{\left[\frac{Sc}{Pr}\right]^{2/3} - x_A\frac{\tilde{C}_{pA}}{\tilde{C}_{p\,mix}} - x_{AS}\left[\left(\frac{Sc}{Pr}\right)^{2/3} - \frac{\tilde{C}_{pA}}{\tilde{C}_{p\,mix}}\right]\right\}}{1 - x_{AS}} \quad \ldots\ldots(36)$$

Now since an air-water vapor mixture $\left[\frac{Sc}{Pr}\right]^{2/3} \approx \frac{\tilde{C}_{pB}}{\tilde{C}_{p\,mix}}$

$$\frac{\partial T}{\partial Z} = (k_{x,loc}a)\,\frac{V(T_s - T)}{LW_B(1 - x_{AS})}\left[\frac{Sc}{Pr}\right]^{2/3}$$

Separating Variables:

$$\frac{dT}{T - T_s} = -(k_{x,loc}a)\,\frac{V\left[\frac{Sc}{Pr}\right]^{2/3}}{LW_B(1 - x_{AS})}\,dz$$

Integrating between the entrance of the bed and some point Z assuming $(k_{x,loc}a)$ to be a function of time only:

$$\ln\frac{T - T_s}{T_1 - T_s} = -(k_{x,loc}a)\,\frac{V\left[\frac{Sc}{Pr}\right]^{2/3}}{W_B(1 - x_{AS})}\left(\frac{Z}{L}\right)$$

Or:

$$\frac{T - T_s}{T_1 - T_s} = e^{-(k_{x,loc}a)\,\frac{V\left[Sc/Pr\right]^{2/3}}{W_B(1 - x_{AS})}\left(\frac{Z}{L}\right)} \qquad \ldots\ldots (37)$$

Where: T_1 = temperature at the entrance to the bed, ^{o}R

Rearranging the temperature distribution in the bed is obtained:

$$T - T_1 = (T_s - T_1)\left[1 - e^{-(k_{x,loc}a)\,\frac{\left[Sc/Pr\right]^{2/3} V}{W_B(1 - x_{AS})}\left(\frac{Z}{L}\right)}\right] \qquad \ldots\ldots (38)$$

Now in order to obtain the concentration in the bed Eq. (8), page 20 is reduced to:

$$-\frac{Px_A m_h}{RT\rho_w V}\frac{\partial M}{\partial \tau} = (k_{x,loc}a)\frac{x_{AS}-x_A}{1-x_{AS}} - \frac{N_B}{(1-x_A)^2}\frac{\partial x_A}{\partial Z} \qquad \ldots.(39)$$

Substituting Eq. (20) into Eq. (39):

$$\frac{\partial x_A}{\partial Z} = \frac{(1-x_A)^2}{N_B}(k_{x,loc}a)\frac{x_{AS}-x_A}{1-x_{AS}}\left[1 - \frac{Px_A M_A}{RT\rho_w V^2}\right] \qquad \ldots.(40)$$

The last term in Eq. (40) is small compared to one, thus Eq. (40) becomes:

$$\frac{\partial x_A}{\partial Z} = \frac{(1-x_A)^2}{N_B}(k_{x,loc}a)\frac{x_{AS}-x_A}{1-x_{AS}} \qquad \ldots.(41)$$

With the assumptions, Eq. (25) reduces to: $\tilde{C}_{pB}(T-T_s) - \frac{x_{AS}-x_A}{1-x_{AS}} H_{fg} = 0$

Or
$$\frac{x_{AS}-x_A}{1-x_{AS}} = -\frac{\tilde{C}_{pB}}{H_{fg}}(T_s - T) \qquad \ldots.(42)$$

Substituting Eq. (37) into Eq. (42):

$$\frac{x_{AS}-x_A}{1-x_{AS}} = \frac{\tilde{C}_{pB}}{H_{fg}}(T_1 - T_s)e^{-(k_{x,loc}a)\frac{[Sc/Pr]^{2/3}V}{W_B(1-x_{AS})}\frac{Z}{L}} \qquad \ldots(43)$$

Now substituting Eq. (43) into Eq. (41):

$$\frac{\partial x_A}{\partial Z} = \frac{(k_{x,loc}a)}{W_B}\frac{\tilde{C}_{pB}}{H_{fg}}(T_1 - T_s)e^{-(k_{x,loc}a)\frac{[Sc/Pr]^{2/3}V}{W_B(1-x_{AS})}\frac{Z}{L}}$$

Integrating we get:

$$x_A - x_{Al} = \frac{\tilde{C}_{pB}(T_l - T_s)L(1 - x_{AS})}{H_{fg}[Sc/Pr]^{2/3}} e^{-(k_{x,loc}a)} \frac{[Sc/Pr]^{2/3} V}{W_B(1 - x_{AS})} \frac{Z}{L} \quad ...(44)$$

Where: x_{Al} = concentration at entrance to bed.

Equation (44) gives the concentration distribution in the free stream as a function of bed depth.

The drying rate can now be determined by substituting Eq. (43) into Eq. (20):

$$\frac{\partial M}{\partial \tau} = (k_{x,loc}a) \frac{M_A V \tilde{C}_{pB}}{m_h H_{fg}} (T_s - T_l) e^{-(k_{x,loc}a) \frac{[Sc/Pr]^{2/3} V}{W_B(1 - x_{AS})} \frac{Z}{L}} \quad ...(45)$$

Integration of Simplified Equations. Now by substituting Eq. (35) into Eq. (45) and dividing both sides by M_i we are prepared to integrate and obtain the drying time during the initial adjustment and constant rate period.

$$\frac{\partial(M/M_i)}{\partial \tau} = \frac{g_c A_o^2 \rho M_A \tilde{C}_{pB} Sc^{-2/3} \Delta P_i}{m_h H_{fg} L W_B M_B^2 M_i Gr} \left[\frac{4m_h}{\pi n \ell}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right]^{1/2} \left[1 - \right.$$

$$\left. \frac{m_h}{V}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right]\left[0.69 \frac{M}{M_i} + 0.31\right](T_s - T_l) \exp -$$

$$\left\{\frac{g_c \rho A_o^2 \Delta P_i (Pr)^{-2/3} Z}{L W_B^2 M_B^2 Gr L (1 - x_{AS})} \left[\frac{4m_h}{\pi n \ell}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right]^{1/2}\right.$$

$$\left[1 - \frac{m_h}{V}\left(\frac{1}{\rho_h} + \frac{M}{\rho_w}\right)\right]\left[0.69 \frac{M}{M_i} + 0.31\right] \quad .... (46)$$

Let:

$$C_1 = \frac{g_c \rho \; M_A \tilde{C}_{pB} \Delta P_i (T_s - T_l) A_o^2 Sc^{-2/3}}{m_h \; H_{fg} \; L \; W_B \; M_B^2 \; M_i \; Gr}$$

$$C_2 = \frac{4 m_h}{\pi \; n \; \ell \; \rho_h}$$

$$C_3 = \frac{4 m_h M_i}{\pi \; n \; \ell \; \rho_w}$$

$$C_4 = \frac{g_c \rho \; Pr^{-2/3} \Delta P_i A_o^2}{W_B^2 M_B^2 \; Gr \; L \; (1 - x_{AS})} \left(\frac{Z}{L}\right)$$

$$C_5 = 1 - \frac{m_h}{V \; \rho_h}$$

$$C_6 = \frac{m_h M_i}{\rho_w V}$$

Thus Eq. (46) becomes:

$$\frac{\partial (M/M_i)}{\partial \tau} = C_1 \left[C_2 + C_3 \frac{M}{M_i}\right]^{1/2} \left[0.69 \frac{M}{M_i} + 0.31\right] \left[C_5 - C_6 \frac{M}{M_i}\right]$$

$$\exp \left\{ - C_4 \left[C_2 + C_3 \frac{M}{M_i}\right]^{1/2} \left[C_5 - C_6 \frac{M}{M_i}\right] \left[0.69 \frac{M}{M_i} + 0.31\right] \right. \quad ..(47)$$

Separating Variables:

$$d\tau = \frac{\exp \left\{ C_4 \left[C_2 + C_3 \frac{M}{M_i}\right]^{1/2} \left[0.69 \frac{M}{M_i} + 0.31\right] \left[C_5 - C_6 \frac{M}{M_i}\right] d\left(\frac{M}{M_i}\right) \right.}{C_1 \left[C_2 + C_3 \frac{M}{M_i}\right]^{1/2} \left[0.69 \frac{M}{M_i} + 0.31\right] \left[C_5 - C_6 \frac{M}{M_i}\right]} \quad ..(48)$$

The exponential term can be expanded into an infinite series. Expanding, neglecting all terms after the first four, since these are negligible when compared to the first term. Integrating between $M/M_i = 1$ and M_c/M_i, the following equations for the drying time between the initial and the critical moisture content result:

For $C_2 < 0.45\, C_3$ and $C_2C_6 < -C_3C_5$

$$\tau_c = \frac{1.45}{C_1\left[\frac{C_5}{C_6} + 0.45\right]}\left\{\frac{2}{C_6\sqrt{0.45\, C_3 - C_2}}\left[\tan^{-1}\sqrt{\frac{\left(C_3\frac{M_c}{M_i}+C_2\right)}{0.45\, C_3 - C_2}}\right.\right.$$

$$\left. - \tan^{-1}\sqrt{\frac{C_3 + C_2}{0.45\, C_3 - C_2}}\right] - \frac{1}{\sqrt{C_6}\sqrt{-C_3C_5 - C_2C_6}}\left[\tan^{-1}\sqrt{\frac{C_3\frac{M_c}{M_i}+C_2}{-C_3C_3-C_2C_6}}\right.$$

$$\left. - \tan^{-1}\sqrt{\frac{C_3 + C_2}{-C_3C_5 - C_2C_6}}\right] + \frac{C_4}{C_1}\left[\frac{M_c}{M_i} - 1\right] + \frac{0.69\, C_4^2 C_6}{C_1C_3}\sqrt{\left(C_3\frac{M_c}{M_i}+C_2\right)^3}$$

$$\left\{\frac{15\, C_3^2\left(\frac{M_c}{M_i}\right)^2 - 12\, C_3C_2\frac{M_c}{M_i} + 8C_2^2}{105\, C_3^2} + \left(\frac{C_5}{C_6} - 0.45\right)\frac{3C_3\frac{M_c}{M_i} - 2C_2}{15C_3}\right.$$

$$\left. + \frac{0.15C_5}{C_6}\right\} - \frac{0.69C_4^2C_6}{C_1C_3}\sqrt{(C_3+C_2)^3}\left\{\frac{15C_3^2 - 12C_3C_2 + 8C_2^2}{105\, C_3^2}\right.$$

$$\left. + \left(\frac{C_5}{C_6} - 0.45\right)\frac{3C_3 - 2C_2}{15C_3} + \frac{0.15C_5}{C_6}\right\} + \frac{C_4^3C_2}{6C_1}\left\{0.095\, C_6^2\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right]\right.$$

$$+\left[0.107C_6^2 - 0.238C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right] + \left[0.159C_5^2 - 0.285C_5C_6 + 0.096C_6^2\right]$$

$$\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + \left[0.214C_5^2 - 0.096C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right] + 0.096C_5^2\left[\frac{M_c}{M_i} - 1\right]$$

$$+ \frac{C_4^3 C_3}{6C_1}\left\{0.079C_6^2\left[\left(\frac{M_c}{M_i}\right)^6 - 1\right] + \left[0.085C_6^2 - 0.190C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right]\right.$$

$$+ \left[0.119C_5^2 - 0.214C_5C_6 + 0.024C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right] + \left[0.143C_5^2 - 0.064C_5C_6\right]$$

$$\left.\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + 0.048C_5^2\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right]\right\} \qquad \ldots\ldots(49a)$$

for $C_2 > 0.45C_3$ and $C_2C_6 > -C_3C_5$

$$\tau_c = \frac{1.45}{C_1\left[\frac{C_5}{C_6} + 0.45\right]}\left\{\frac{1}{C_6\sqrt{C_2 - 0.45C_3}}\ln\left|\frac{\sqrt{C_3\frac{M_c}{M_i} + C_2} - \sqrt{C_2 - 0.45C_3}}{\sqrt{C_3\frac{M_c}{M_i} + C_2} + \sqrt{C_2 - 0.45C_3}}\right|\right.$$

$$\left. - \frac{1}{C_6\sqrt{C_2 - 0.45C_3}}\ln\left|\frac{\sqrt{C_3 + C_2} - \sqrt{C_2 - 0.45C_3}}{\sqrt{C_3 + C_2} + \sqrt{C_2 + 0.45C_3}}\right|\right\} - \frac{1}{\sqrt{C_6}\sqrt{C_2C_6 + C_3C_5}}$$

$$\left\{\ln\left|\frac{\sqrt{C_6\left(C_3\frac{M_c}{M_i} + C_2\right)} - \sqrt{C_2C_6 + C_3C_5}}{\sqrt{C_6\left(C_3\frac{M_c}{M_i} + C_2\right)} + \sqrt{C_2C_6 + C_3C_5}}\right| - \ln\left|\frac{\sqrt{C_6(C_3 + C_2)} - \sqrt{C_2C_6 + C_3C_5}}{\sqrt{C_6(C_3 + C_2)} + \sqrt{C_2C_6 + C_3C_5}}\right|\right\}$$

$$+\frac{C_4}{C_1}\left[\frac{M_c}{M_i}-1\right]+\frac{0.69C_4^2C_6}{C_1C_3}\sqrt{\left(C_3\frac{M_c}{M_i}+C_2\right)^3}\left\{\frac{15C_3^2\left(\frac{M_c}{M_i}\right)^2-12C_3C_2\frac{M_c}{M_i}+8C_2^2}{105C_3^2}\right.$$

$$\left.+\left(\frac{C_5}{C_6}-0.45\right)\frac{3C_3\frac{M_c}{M_i}-2C_2}{15C_3}+\frac{0.15C_5}{C_6}\right\}-\frac{0.69C_4^2C_6}{C_1C_3}\sqrt{\left(C_3+C_2\right)^3}$$

$$\left\{\frac{15C_3^2-12C_3C_2+8C_2^2}{105C_3^2}+\left(\frac{C_5}{C_6}-0.45\right)\frac{3C_3-2C_2}{15C_3}+\frac{0.15C_5}{C_6}\right\}$$

$$+\frac{C_4^3C_2}{6C_1}\left\{0.095C_6^2\left[\left(\frac{M_c}{M_i}\right)^5-1\right]+\left[0.107C_6^2-0.238C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^4-1\right]\right.$$

$$+\left[0.159C_5^2-0.285C_5C_6+0.096C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^3-1\right]+\left[0.214C_5^2\right.$$

$$\left.-0.096C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^2-1\right]+0.096C_5^2\left[\frac{M_c}{M_i}-1\right]+\frac{C_4^3C_3}{6C_1}$$

$$\left\{0.079C_6^2\left[\left(\frac{M_c}{M_i}\right)^6-1\right]+\left[0.085C_6^2-0.190C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^5-1\right]\right.$$

$$+\left[0.119C_5^2-0.214C_5C_6+0.024C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^4-1\right]+\left[0.143C_5^2\right.$$

$$\left.\left.-0.064C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^3-1\right]+0.048C_5^2\left[\left(\frac{M_c}{M_i}\right)^2-1\right]\right\} \qquad \ldots.(49b)$$

Where τ_c = elapsed time between initial moisture content and critical moisture content minutes.

M_c = critical moisture content, lbm moisture/lbm dry material.

A comparison of the drying time predicted by Eq. (49) and the experimental measured values are given in Table 4, page 127.

The Rate of Drying During the Falling Rate Period

The following approximate method as suggested by Perry[3] gives a fairly accurate expression for the drying time during this region of drying.

The drying rate can be expressed as follows:

$$\frac{\partial M}{\partial \tau} = -\beta(M - M_e) \qquad \ldots\ldots(50)$$

Where: β = slope of straight line approximating the falling rate period.

M_e = equilibrium moisture content, lbm moisture/lbm dry material.

Since: $\left(\frac{\partial M}{\partial \tau}\right)_c = -\beta(M_c - M_e)$

$$\beta = \frac{-(\partial M/\partial \tau)_c}{M_c - M_e} \qquad \ldots\ldots(51)$$

Where: $\left(\frac{\partial M}{\partial \tau}\right)_c$ = rate of drying at the critical moisture content, lbm moisture/lbm dry material, minute.

Substituting Eq. (51) into Eq. (50):

$$\frac{\partial M}{\partial \tau} = \left(\frac{\partial M}{\partial \tau}\right)_c \frac{M - M_e}{M_c - M_e}$$

Let:

$$r = \frac{M - M_e}{M_c - M_e}$$

$$\frac{\partial r}{\partial \tau} = \frac{1}{M_c - M_e} \frac{\partial M}{\partial \tau}$$

Thus:

$$\frac{\partial r}{\partial \tau} = \left(\frac{\partial M}{\partial \tau}\right)_c \frac{r}{M_c - M_e}$$

Separating variables and changing to total differentials:

$$\frac{dr}{r} = \left(\frac{\partial M}{\partial \tau}\right)_c \frac{d\tau}{M_c - M_e}$$

Integrating:

$$- \ln r = \left(\frac{\partial M}{\partial \tau}\right)_c \frac{\tau - \tau_c}{M_c - M_e} \qquad \ldots\ldots(52)$$

Under normal industrial drying conditions, the drying temperature is above 180°F and thus $M_e \simeq 0$. Thus: $r \approx \frac{M}{M_c}$

$$\tau - \tau_c = \ln \frac{M_c}{M} \frac{M_c}{(\partial M/\partial \tau)_c} \qquad \ldots\ldots(53)$$

The material will be considered as dry when $M = M_e$; therefore,

$$\tau - \tau_c = \ln \left(\frac{M_c}{M_e}\right) \frac{M_c}{(\partial M/\partial \tau)_c} \qquad (54)$$

Total Drying Time

By using Eq. (49) and Eq. (54) the total drying time can be predicted if the critical moisture is known.

Combining Eqs. (49) and (54), the equation for the total drying time becomes:

for: $C_2 < 0.45\,C_3$ and $C_2C_6 < -C_3C_5$

$$\tau = \frac{1.45}{C_1\left[\frac{C_5}{C_6} + 0.45\right]}\left\{\frac{2}{C_6\sqrt{0.45C_3 - C_2}}\left[\tan^{-1}\sqrt{\frac{(C_3\frac{M_c}{M_i} + C_2)}{0.45C_3 - C_2}}\right.\right.$$

$$\left. - \tan^{-1}\sqrt{\frac{C_3 + C_2}{0.45C_3 - C_2}}\right] - \frac{1}{\sqrt{C_6}\sqrt{-C_3C_5 - C_2C_6}}\left[\tan^{-1}\sqrt{\frac{C_3\frac{M_c}{M_i} + C_2}{-C_3C_5 - C_2C_6}}\right.$$

$$\left. - \tan^{-1}\sqrt{\frac{C_3 + C_2}{-C_3C_5 - C_2C_6}}\right] + \frac{C_4}{C_1}\left[\frac{M_c}{M_i} - 1\right] + \frac{0.69C_4^2C_6}{C_1C_3}\sqrt{(C_3\frac{M_c}{M_i} + C_2)^3}$$

$$\left\{\frac{15C_3^2\left(\frac{M_c}{M_i}\right)^2 - 12C_3C_2\frac{M_c}{M_i} + 8C_2^2}{105C_3^2} + \left(\frac{C_5}{C_6} - 0.45\right)\frac{3C_3\frac{M_c}{M_i} - 2C_2}{15C_3}\right.$$

$$\left. + \frac{0.15C_5}{C_6}\right\} - \frac{0.69C_4^2C_6}{C_1C_3}\sqrt{(C_3 + C_2)^3}\left\{\frac{15C_3^2 - 12C_3C_2 + 8C_2^2}{105C_3^2}\right.$$

$$\left. + \left(\frac{C_5}{C_6} - 0.45\right)\frac{3C_3 - 2C_2}{15C_3} + \frac{0.15C_5}{C_6}\right\} + \frac{C_4^3C_2}{6C_1}\left\{0.095C_6^2\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right]\right.$$

$$+ \left[0.107C_6^2 - 0.238C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right] + \left[0.159C_5^2 - 0.285C_5C_6 + 0.096C_6^2\right]$$

$$\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + \left[0.214C_5^2 - 0.096C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right] + 0.096C_5^2\left[\frac{M_c}{M_i} - 1\right]$$

$$+\frac{C_4^3 C_3}{6C_1}\left\{0.079C_6^2\left[\left(\frac{M_c}{M_i}\right)^6-1\right]+\left[0.085C_6^2-0.190C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^5-1\right]\right.$$

$$+\left[0.119C_5^2-0.214C_5C_6+0.024C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^4-1\right]+\left[0.143C_5^2\right.$$

$$\left.\left.-0.064C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^3-1\right]+0.048C_5^2\left[\left(\frac{M_c}{M_i}\right)^2-1\right]\right\}+\ln\left(\frac{M_c}{M_e}\right)\frac{M_c}{(\partial M/\partial\tau)_c}$$

.....(55a)

and for: $C_2 > 0.45C_3$ and $C_2C_6 > -C_3C_5$

$$\tau_c=\frac{1.45}{C_1\left[\frac{C_5}{C_6}+0.45\right]}\left\{\frac{1}{C_6\sqrt{C_2-0.45C_3}}\ln\left|\frac{\sqrt{C_3\frac{M_c}{M_i}+C_2}-\sqrt{C_2-0.45C_3}}{\sqrt{C_3\frac{M_c}{M_i}+C_2}+\sqrt{C_2-0.45C_3}}\right|\right.$$

$$\left.-\frac{1}{C_6\sqrt{C_2-0.45C_3}}\ln\left|\frac{\sqrt{C_3+C_2}-\sqrt{C_2-0.45C_3}}{\sqrt{C_3+C_2}+\sqrt{C_2-0.45C_3}}\right|\right\}-\frac{1}{\sqrt{C_6}\sqrt{C_2C_6+C_3C_5}}$$

$$\left\{\ln\left|\frac{\sqrt{C_6(C_3\frac{M_c}{M_i}+C_2)}-\sqrt{C_2C_6+C_3C_5}}{\sqrt{C_6(C_3\frac{M_c}{M_i}+C_2)}+\sqrt{C_2C_6+C_3C_5}}\right|-\ln\left|\frac{\sqrt{C_6(C_3+C_2)}-\sqrt{C_2C_6+C_3C_5}}{\sqrt{C_6(C_3+C_2)}+\sqrt{C_2C_6+C_3C_5}}\right|\right\}$$

$$+\frac{C_4}{C_1}\left[\frac{M_c}{M_i}-1\right]+\frac{0.69C_4^2C_6}{C_1C_3}\sqrt{\left(C_3\frac{M_c}{M_i}+C_2\right)^3}$$

$$\left\{\frac{15C_3^2\left(\frac{M_c}{M_i}\right)^2-12C_3C_2\frac{M_c}{M_i}+8C_2^2}{105C_3^2}+\left(\frac{C_5}{C_6}-0.45\right)\frac{3C_3\frac{M_c}{M_i}-2C_2}{15C_3}+\frac{0.15C_5}{C_6}\right\}$$

$$- \frac{0.69C_4^2C_6}{C_1C_3}\sqrt{(C_3 + C_2)^3}\left\{\frac{15C_3^2 - 12C_3C_2 + 8C_2^2}{105C_3^2} + \left(\frac{C_5}{C_6} - 0.45\right)\right.$$

$$\left.\frac{3C_3 - 2C_2}{15C_3} + \frac{0.15C_5}{C_6}\right\} + \frac{C_4^3C_2}{6C_1}\left\{0.095\ C_6^2\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right]\right.$$

$$+ \left[0.107C_6^2 - 0.238C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right] + \left[0.159C_5^2 - 0.285C_5C_6\right.$$

$$\left. + 0.096C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + \left[0.214C_5^2 - 0.096C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right]$$

$$+ 0.096C_5^2\left[\frac{M_c}{M_i} - 1\right] + \frac{C_4^3C_3}{6C_1}\left\{0.079C_6^2\left[\left(\frac{M_c}{M_i}\right)^6 - 1\right] + \left[0.085C_6^2\right.\right.$$

$$\left. - 0.190C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right] + \left[0.119C_5^2 - 0.214C_5C_6 + 0.024C_6^2\right]$$

$$\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right] + \left[0.143C_5^2 - 0.064C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + 0.048C_5^2$$

$$\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right] + \ln\left(\frac{M_c}{M_e}\right)\frac{M_c}{(\partial M/\partial\tau)_c} \qquad \ldots.(55b)$$

When no value for the critical moisture is available, a critical moisture content of between 60 to 90 per cent may be assumed with reasonable accuracy.

Typical Computations

Experimental Data: Run No.: 106

Date: 7-22-64

Material: Coastal Bermuda grass

Length at cutting = 15 ins., approximately

Particle Length = 1 in.

P (atm) = 29.16 in. Hg.

Flow Rate = 365 cfm

ΔP(wet sample) = 1.805 ins. H_2O

ΔP(dry sample) = 0.813 ins. H_2O

T_d = 82° F

T_w = 75° F

T_{duct} = 270° F

Bed Thickness = 4 ins.

Weight dry material = 0.84 lbm

Initial moisture content = 2.57

Area of Sample = 113 $in.^2$

n = 48,000 particles/lbm

ρ_h = 65 $lbm/ft.^3$

$\frac{M_c}{M_i}$ = 0.311

Pressure Drop Calculations: From Eq. (30) the average diameter of the dry particle is:

$$D_p = \left[\frac{4m_h}{\pi n \ell \rho_h}\right]^{1/2}$$

$$D_p = \left[\frac{4 \times 0.84 \text{ lbm} \times 12 \frac{\text{in}}{\text{ft.}}}{\pi \times 48{,}000 \frac{\text{pt.}}{\text{lbm}} \times 0.84 \text{ lbm} \times 1\text{in} \times 65 \frac{\text{lbm}}{\text{ft}^3}} \right] = 2.21 \times 10^{-3} \text{ ft.}$$

Neglecting the mass of water vapor in the air, the density of the air using the ideal gas equation is:

$$\rho = \frac{P}{RT} = \frac{14.3 \frac{\text{lbf}}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2}}{15.35 \frac{\text{ft-lbf}}{\text{lbm } ^\circ\text{R}} \times 730^\circ\text{R}} = 0.0536 \frac{\text{lbm air}}{\text{ft}^3}$$

The mass flow rate is:

$$G = \rho Q = 0.0536 \frac{\text{lbm air}}{\text{ft}^3} \times 365 \frac{\text{ft}^3}{\text{min}} = 19.6 \frac{\text{lbm air}}{\text{min}}$$

From Eq. (28) the porosity of the dry bed is:

$$X = 1 - \frac{0.84 \text{ lbm}}{0.262 \text{ ft}^3} \quad \frac{1}{65 \frac{\text{lbm}}{\text{ft}^3}} = 0.9506$$

The modified Reynolds number is from Eq. (26)

$$Re = \frac{D_p G}{\mu X}$$

At: $T = 270^\circ\text{F}$, $\mu = 1.542 \times 10^{-5} \frac{\text{lbm}}{\text{ft sec}}$

$$Re = \frac{2.21 \times 10^{-3} \text{ dt} \times 19.6 \frac{\text{lbm air}}{\text{min}}}{1.542 \times 10^{-5} \frac{\text{lbm}}{\text{ft sec}} \times 0.9506 \times 60 \frac{\text{sec}}{\text{min}} \times 0.7854 \text{ ft}^2}$$

$$Re = 62.5$$

The corresponding friction factor from Figure 12 is 0.530, thus from Eq. (27) the pressure drop across the dry bed is:

$$\Delta P_d = \frac{f L G^2}{2 g_c D_p X \rho}$$

$$\Delta P_d = 0.820 \text{ in } H_2O$$

The experimental measured pressure drop is:

$$\Delta P_d = 0.813 \text{ in } H_2O$$

Total Drying Time: The constants in Eq. (55) must first be determined. From page

$$C_1 = \frac{g_c \rho A_o^2 M_A \tilde{C}_{PB} \Delta P_i (T_s - T_l) Sc^{-2/3}}{m_h H_{fg} L N_B M_B^2 M_i Gr}$$

$$= \frac{g_c A_o^2 C_{PS} \Delta P_i (T_s - T_l) Sc^{-2/3}}{m_h h_{fg} L Q M_i Gr}$$

At $T = 270^oF$, $Sc = 0.70$, from the high temperature psychometric chart, $T_s = 107^oF$, and $Gr = 1.0$ from Figure 11.

$$C_1 = -125.2 \frac{1}{\text{ft. min.}}$$

Now: $C_2 = \frac{4m_h}{\pi n \ell \rho_h} = D_p^2$

$$C_2 = 2.21^2 \times 10^{-6}\ \text{ft}^2 = 4.89 \times 10^{-6}\ \text{ft}^2$$

And: $C_3 = \frac{4m_h M_i}{\pi n \ell \rho_w} = C_2 \frac{\rho_h}{\rho_w} M_i$

$$C_3 = 3.89 \times 10^{-6}\ \text{ft}^2 \times \frac{65.0 \frac{\text{lbm hay}}{\text{ft}^2}}{62.4 \frac{\text{lbm } H_2O}{\text{ft}^3}} \times 2.57 \frac{\text{lbm } H_2O}{\text{lbm hay}}$$

$$C_3 = 13.0 \times 10^{-6}\ \text{ft}^2$$

The next constant is:

$$C_4 = \frac{g_c \rho\, Sc^{-2/3}\, \Delta P_i A_o^2}{L\, W_B^2\, M_B^2\, Gr} (1 + x_{AS})(\frac{Z}{L})$$

Rearranging:

$$C_4 = \frac{g_c Sc^{-2/3}\, \Delta P_i A_o^2}{Q^2 \rho\, Gr\, L} (1 + x_{AS})(\frac{Z}{L})$$

At T_s = 107° F the partial pressure of the water vapor at the surface is 0.117 psi and thus x_{AS} = 0.101. Therefore,

$$C_4 = \frac{32.2 \frac{\text{lbm-ft}}{\text{lbf-sec}^2} \times 0.70^{-2/3} \times 1.8 \text{ in } H_2O \times 5.18 \frac{\text{lbf}}{\text{ft}^2 \text{ in } H_2O} \times 0.7854^2\ \text{ft}^4 \times 1.101 \times 3600 \frac{\text{sec}^2}{\text{min}^2}}{(365)^2 \frac{\text{ft}^6}{\text{min}^2} \times 0.0536 \frac{\text{lbm}}{\text{ft}^3} \times 1.0 \times 0.33\ \text{ft}}$$

$$C_4 = 302.0$$

$$C_5 = 1 - \frac{m_h}{\rho_h V} = 1 - \frac{0.84 \text{ lbm}}{65.0 \frac{\text{lbm}}{\text{ft}^3} \times 0.262 \text{ ft}^3} = 0.9507$$

$$C_6 = \frac{m_h M_i}{\rho_w V} = \frac{0.84 \text{ lbm hay} \times 2.57 \frac{\text{lbm } H_2O}{\text{lbm hay}}}{62.4 \frac{\text{lbm } H_2O}{\text{ft}^3} \times 0.262 \text{ ft}^3} = 0.132$$

From experimental data:

$$\frac{M_c}{M_i} = 0.311$$

The rate of drying at the critical moisture content is determined from Eq. (47).

$$\frac{\partial (M_c/M_i)}{\partial \tau} = C_1 \left[C_2 + C_3 \frac{M_c}{M_i}\right]^{1/2} \left[0.69 \frac{M_c}{M_i} + 0.31\right] \left[C_5 - C_6 \frac{M_c}{M_i}\right] \exp \left\{-C_4 \left[C_2 + C_3 \frac{M_c}{M_i}\right]^{1/2} \left[0.69 \frac{M_c}{M_i} + 0.31\right] \left[C_5 - C_6 \frac{M_c}{M_i}\right]\right.$$

$$\frac{\partial (M_c/M_i)}{\partial \tau} = -125.2 \frac{1}{\text{ft min}} \left[4.89 + 13.0 \times 0.311\right]^{1/2} \times 10^{-3} \text{ft} \left[0.69 \times 0.311 + 0.31\right] \times \left[0.9507 - 0.132 \times 0.311\right] \exp \left\{-302.0 \frac{1}{\text{ft}} \left[4.89 + 13.0 \times 0.311\right]^{1/2} \times 10^{-3} \text{ ft} \times \left[0.69 \times 0.311 + 0.31\right] \times \left[0.9507 - 0.132 \times 0.311\right] =$$

$$\frac{\partial (M_c/M_i)}{\partial \tau} = 0.116 \frac{1}{\text{min}}$$

Since $C_2 < 0.45\ C_3$ and $C_2C_6 < -\ C_3C_5$ the total drying time is determined by the following equation:

$$\tau = \frac{1.45}{C_1\left[\frac{C_5}{C_6} + 0.45\right]}\left\{\frac{2}{C_6\sqrt{0.45C_3 - C_2}}\left[\tan^{-1}\sqrt{\frac{(C_3\frac{M_c}{M_i} + C_2)}{0.45C_3 - C_2}} - \tan^{-1}\sqrt{\frac{C_3 + C_2}{0.45C_3 - C_2}}\right]\right.$$

$$- \frac{1}{\sqrt{C_6}\sqrt{C_2C_6 + C_3C_5}}\left[\ln\left|\frac{\sqrt{C_6(C_3\frac{M_c}{M_i} + C_2)} - \sqrt{C_2C_6 + C_3C_5}}{\sqrt{C_6(C_3\frac{M_c}{M_i} + C_2)} + \sqrt{C_2C_6 + C_3C_5}}\right|\right.$$

$$\left.\left. - \ln\left|\frac{\sqrt{C_6(C_3 + C_2)} - \sqrt{C_2C_6 + C_3C_5}}{\sqrt{C_6(C_3 + C_2)} + \sqrt{C_2C_6 + C_3C_5}}\right|\right] + \frac{C_4}{C_1}\left[\frac{M_c}{M_i} - 1\right] + \frac{0.69C_4^2C_6}{C_1C_3}$$

$$\sqrt{(C_3\frac{M_c}{M_i} + C_2)^3}\left\{\frac{15C_3^2\left(\frac{M_c}{M_i}\right)^2 - 12C_3C_2\frac{M_c}{M_i} + 8C_2^2}{105\ C_3^2} + \left(\frac{C_5}{C_6} - 0.45\right)\right.$$

$$\left.\frac{3C_3\frac{M_c}{M_i} - 2C_2}{15C_3} + \frac{0.15C_5}{C_6}\right\} - \frac{0.69C_4^2C_6}{C_1C_3}\sqrt{(C_3 + C_2)^3}$$

$$\left\{\frac{15C_3^2 - 12C_3C_2 + 8C_2^2}{105\ C_3^2} + \left(\frac{C_5}{C_6} - 0.45\right)\frac{3C_3 - 2C_2}{15C_3} + \frac{0.15C_5}{C_6}\right\}$$

$$+ \frac{C_4^3C_2}{6C_1}\left\{0.95C_6^2\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right] + \left[0.107C_6^2 - 0.238C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right]\right.$$

$$+\left[0.159C_5^2 - 0.285C_5C_6 + 0.096C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + \left[0.214C_5^2 - 0.096C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right] + 0.096C_5^2\left[\frac{M_c}{M_i} - 1\right]\Bigg\}$$

$$+\frac{C_4^3C_3}{6C_1}\Bigg\{0.079C_6^2\left[\left(\frac{M_c}{M_i}\right)^6 - 1\right] + \left[0.085C_6^2 - 0.190C_5C_6\right]\left[\left(\frac{M_c}{M_i}\right)^5 - 1\right]$$

$$+\left[0.119C_5^2 - 0.214C_5C_6 + 0.024C_6^2\right]\left[\left(\frac{M_c}{M_i}\right)^4 - 1\right] + \left[0.143C_5^2 - 0.064C_5C_6\right]$$

$$\left[\left(\frac{M_c}{M_i}\right)^3 - 1\right] + 0.048C_5^2\left[\left(\frac{M_c}{M_i}\right)^2 - 1\right]\Bigg\} + \ln\left(\frac{M_c}{M_e}\right)\frac{M/M_i}{\left(\frac{\partial M/M_i}{\partial\tau}\right)_c}$$

Substituting values for the constants; assuming $M_e = 3\%$

$$\tau = \frac{1.45}{(-125.2\ \frac{1}{\text{ft min}})\left[\frac{0.9507}{0.132} + 0.45\right]}\Bigg\{\frac{2}{0.132\sqrt{0.45(13.0) - 4.89}\ (10^{-3})\text{ft}}$$

$$\Bigg[\tan^{-1}\sqrt{\frac{(13.0)(0.311) + 4.89}{0.45(13.0) - 4.89}} - \tan^{-1}\sqrt{\frac{13.0 + 4.89}{0.45(13.0) - 4.89}}$$

$$- \frac{1}{\sqrt{0.132}\sqrt{0.132(4.89) + 0.9507(13.0)}\ (10^{-3})\ \text{ft}}$$

$$\Bigg[\ln\left|\frac{\sqrt{0.132[(13.0)(0.311) + 4.89]} - \sqrt{0.132(4.89) + 0.9507(13.0)}}{\sqrt{0.132[(13.0)(0.311) + 4.89]} + \sqrt{0.132(4.89) + 0.9507(13.0)}}\right|$$

$$- \ln\left|\frac{\sqrt{0.132[13.0 + 4.89]} - \sqrt{0.132(4.89) + 0.9507(13.0)}}{\sqrt{0.132[13.0 + 4.89]} + \sqrt{0.132(4.89) + 0.9507(13.0)}}\right|$$

$$+ \frac{302.0\ \frac{1}{\text{ft}}}{-125.2\ \frac{1}{\text{ft min}}}\left[0.311 - 1.0\right] + \frac{0.69'302.0\ \frac{1}{\text{ft}})^2\ (0.132)}{(-125.2\ \frac{1}{\text{ft min}})\ (13.0 \times 10^{-6}\ \text{ft}^2)}$$

$$\sqrt{\left[(13.0)(0.311)+4.89\right]^3}\ (10^{-9})\,ft^3$$

$$\left\{\frac{15(13.0)^2(0.311)^2 - 12(13.0)(4.89)(0.311) + 8(4.89)^2}{105(13.0)^2}\right.$$

$$\left. + \left(\frac{0.9507}{0.132} - 0.45\right)\frac{3(13.0)(0.311) - 2(4.89)}{15(13.0)} + \frac{0.15(0.9507)}{0.132}\right\}$$

$$- \frac{0.69(302.0\ \frac{1}{ft})^2\,(0.132)}{(-125.2\ \frac{1}{ft\ min})\,(13.0 \times 10^{-6} ft^2)}\ \sqrt{(13.0 + 4.89)^3}\ (10^{-9})\,ft^3$$

$$\left\{\frac{15(13.0)^2 - 12(13.0)(4.89) + 8(4.89)^2}{105(13.0)^2} + \left(\frac{0.9507}{0.132} - 0.45\right)\right.$$

$$\left.\frac{3(13.0) - 2(4.89)}{15(13.0)} + \frac{0.15(0.9507)}{0.132}\right\} + \frac{(348.0\ \frac{1}{ft})^3(4.89 \times 10^{-6} ft^2)}{6(-125.2\ \frac{1}{ft\ min})}$$

$$\left\{0.095(0.132)^2\,[(0.311)^5 - 1.0] + [0.107(0.132)^2 - 0.238(0.951)(0.132)]\right.$$

$$[(0.311)^4 - 1.0] + [0.159(0.951)^2 - 0.285(0.951)(0.132)$$

$$+ 0.096(0.132)^2][(0.311)^3 - 1.0] + 0.096(0.951)^2[0.311 - 1.0]$$

$$+ \frac{(302.0\ \frac{1}{ft})^3\,(13.0 \times 10^{-6} ft^2)}{6(-125.2\ \frac{1}{ft\ min})}\left\{0.079(0.132)^2\,[(0.311)^6 - 1.0]\right.$$

$$+ [0.085(0.132)^2 - 0.190(0.951)(0.132)][(0.311)^5 - 1.0]$$

$$+ [0.119(0.951)^2 - 0.214(0.951)(0.132) + 0.024(0.132)^2][(0.311)^4 - 1.0]$$

$$+ [0.143(0.951)^2 - 0.064(0.951)(0.132)][(0.311)^3 - 1.0] + 0.048(0.951)^2$$

$$[(0.311)^2 - 1.0] + \ln[(33.3)(0.80)]\ \frac{0.311}{0.116\ \frac{1}{min}} = 11.81\ min$$

$\tau = 1.82 + 1.65 - 0.16 + 0.80 + 0.15 + 7.55 = 11.81$ minutes

From Figure 29 the experimental drying time is 11.5 minutes.

APPENDIX B

GRAPHICAL REPRESENTATION OF RESULTS

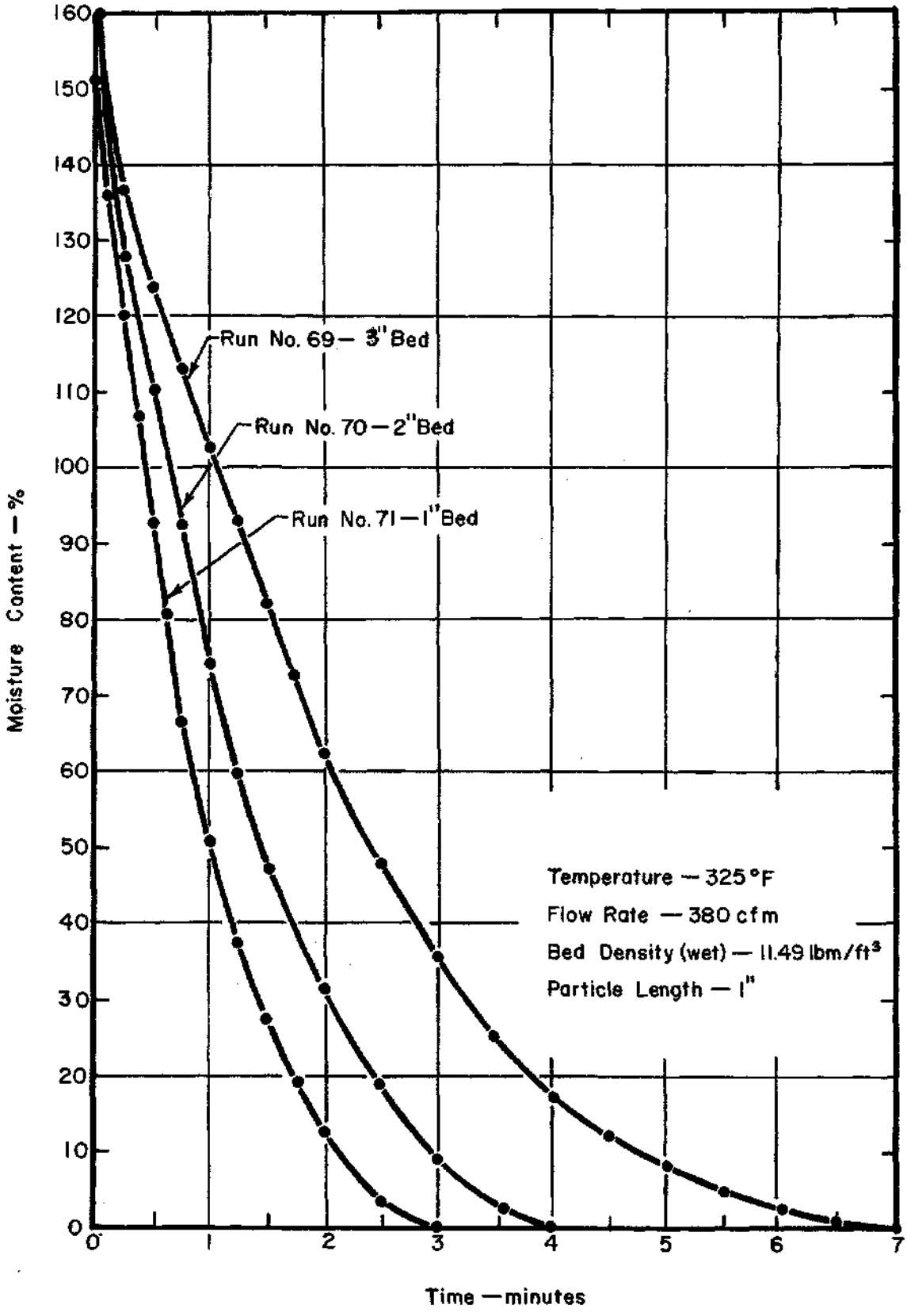


Figure 25. Drying Curves for Various Bed Thicknesses

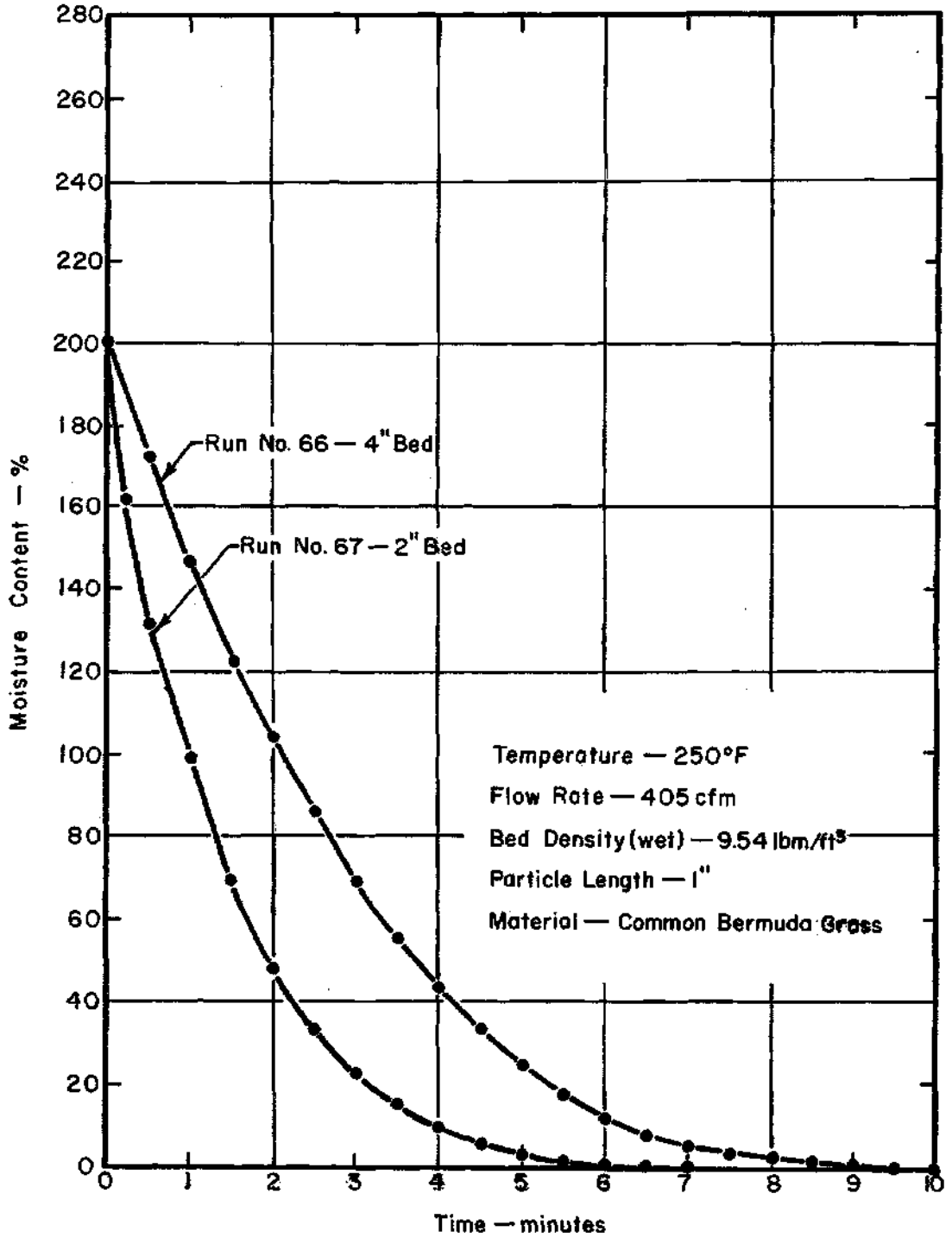


Figure 26. Drying Curves for two Bed Thicknesses for Common Bermuda Grass

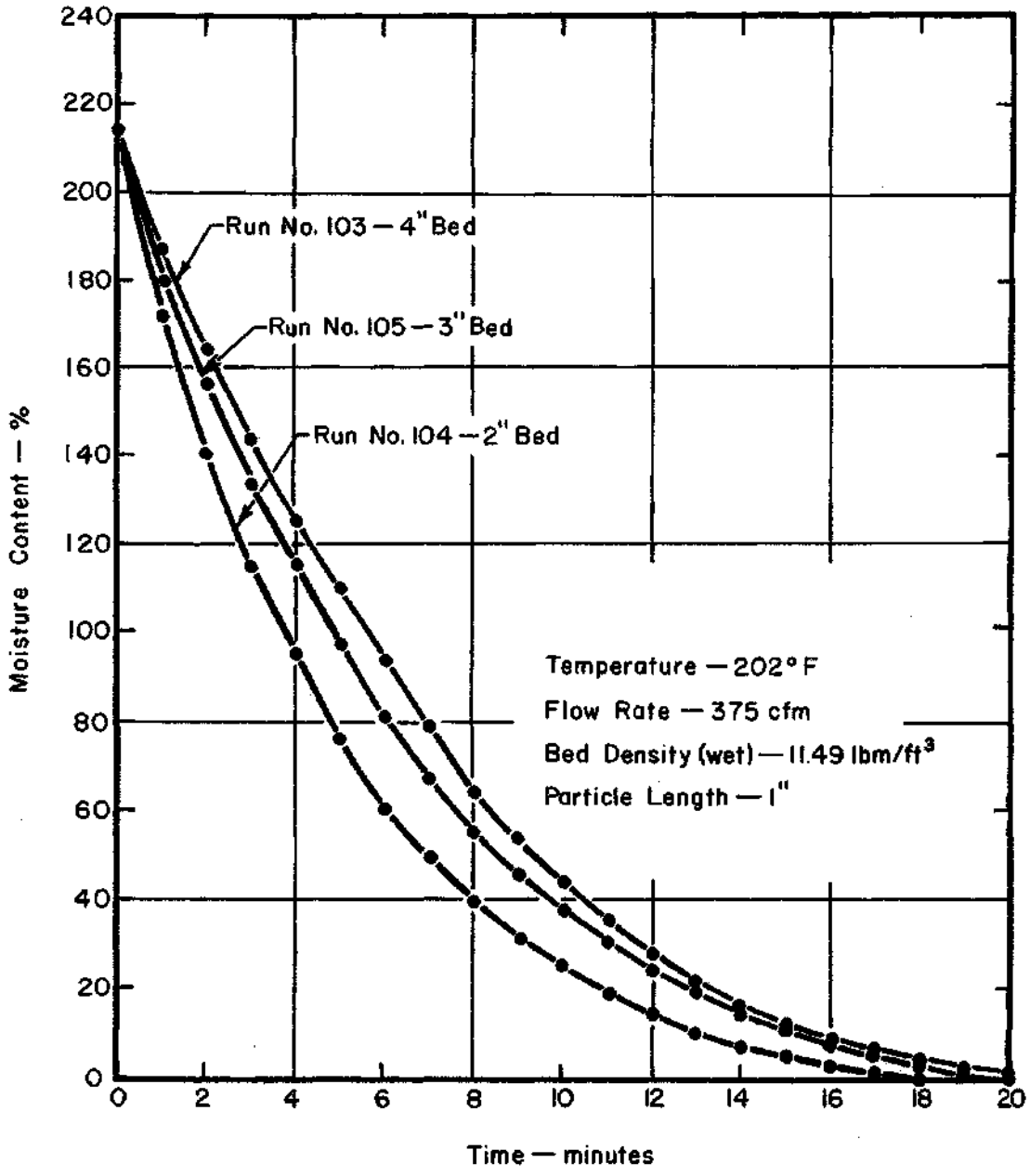


Figure 27. Drying Curves for Various Bed Thicknesses

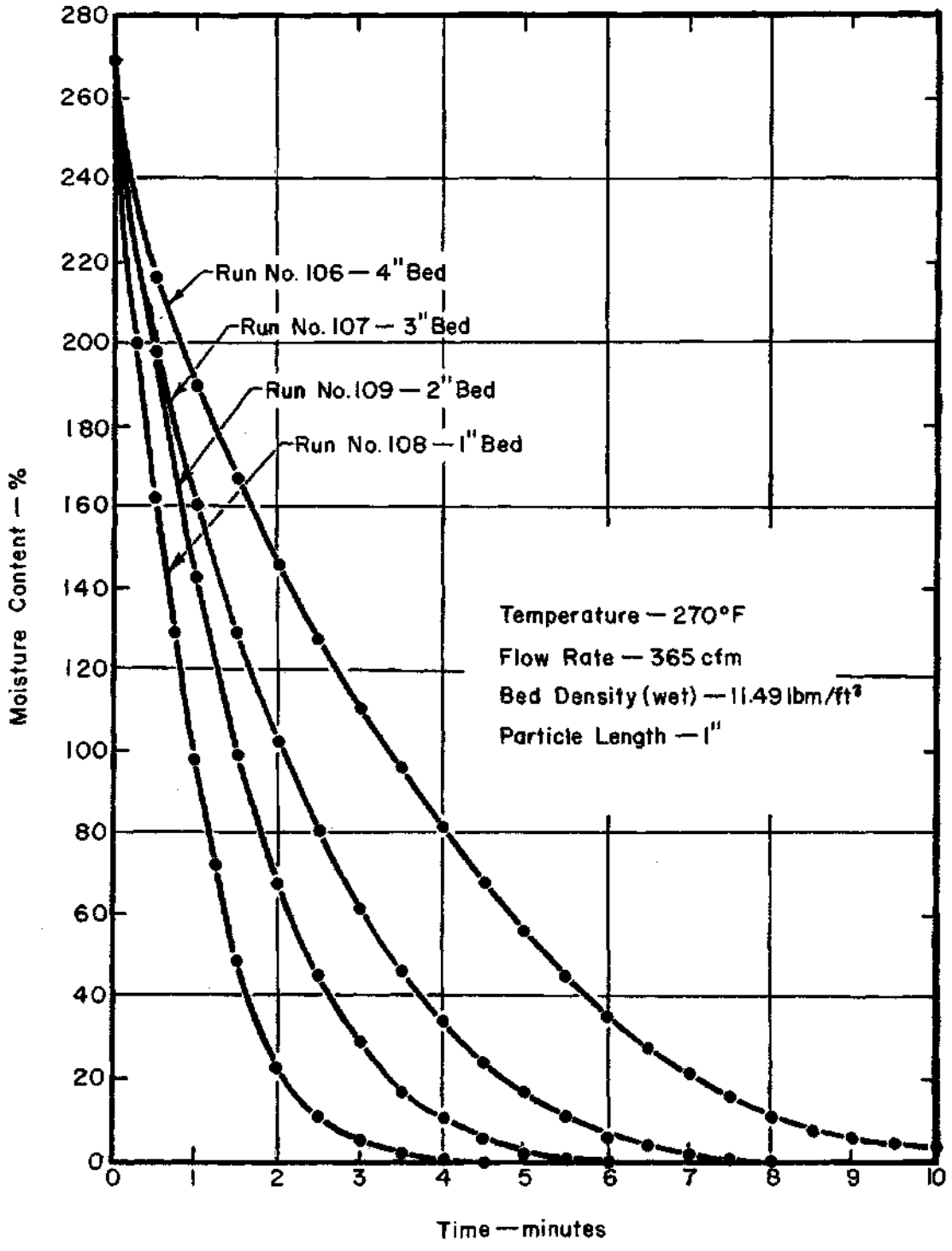


Figure 28. Drying Curves for Various Bed Thicknesses

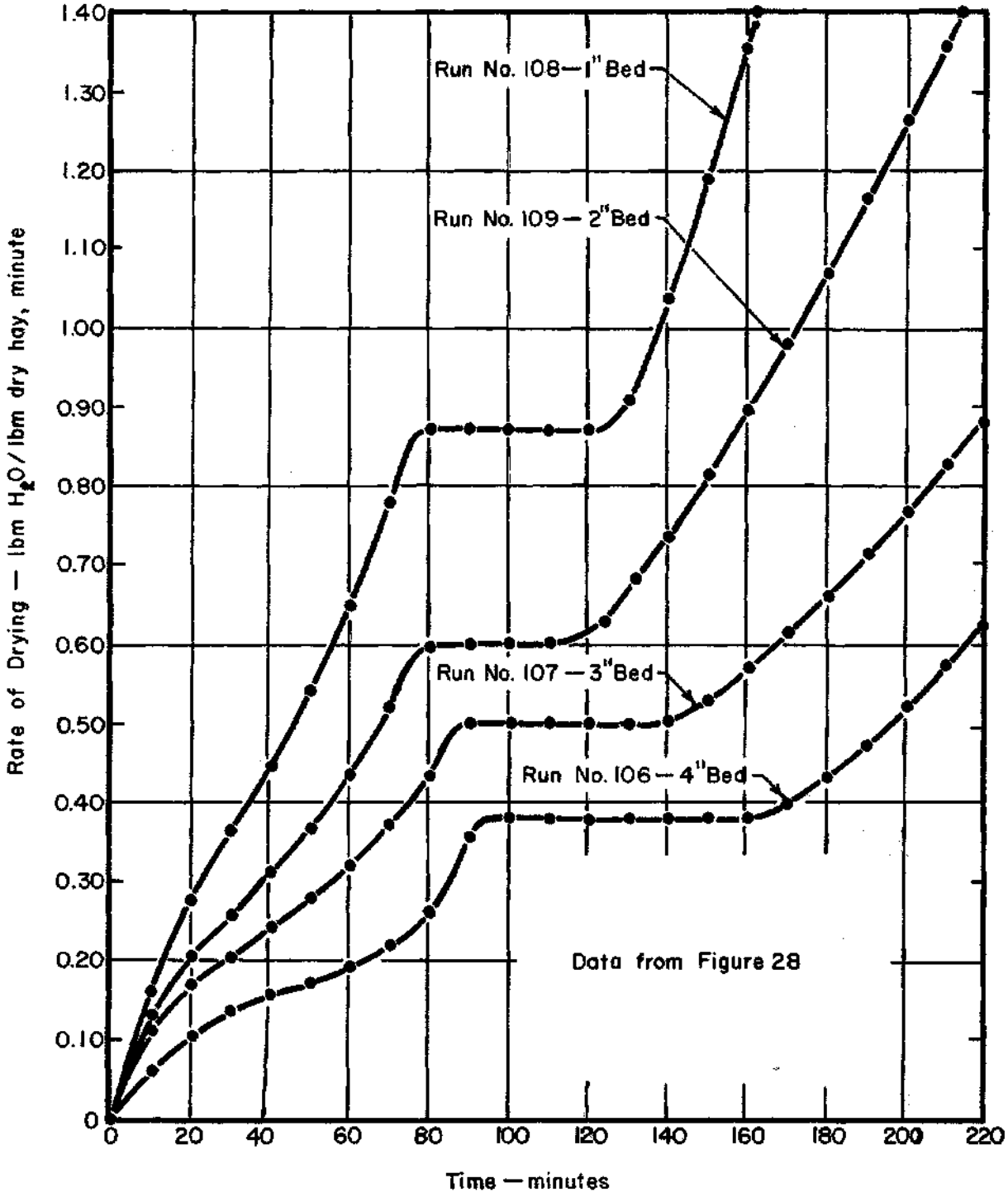


Figure 29. Rate of Drying Curves for Various Bed Thicknesses

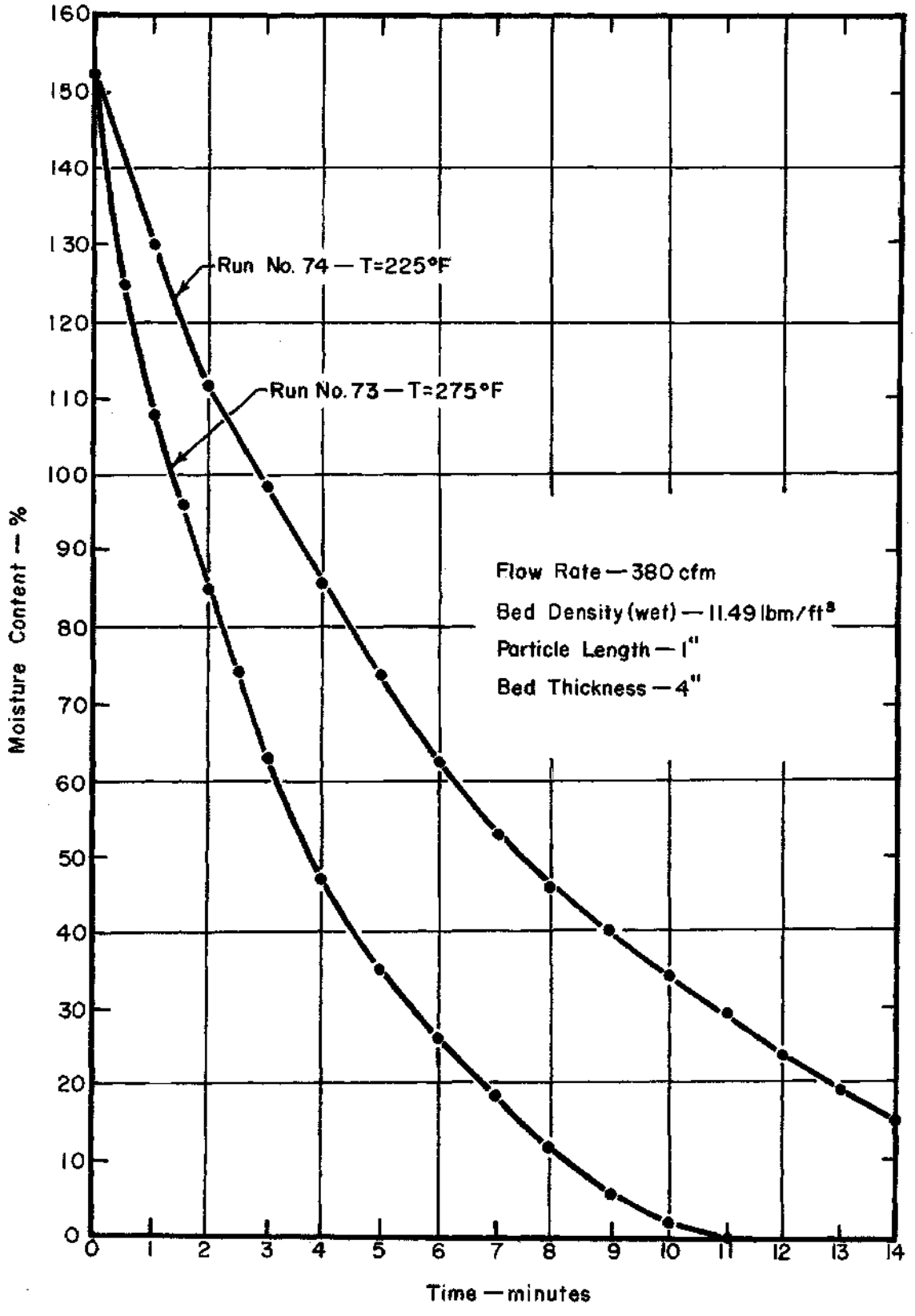


Figure 30a. Drying Curves for Various Temperatures

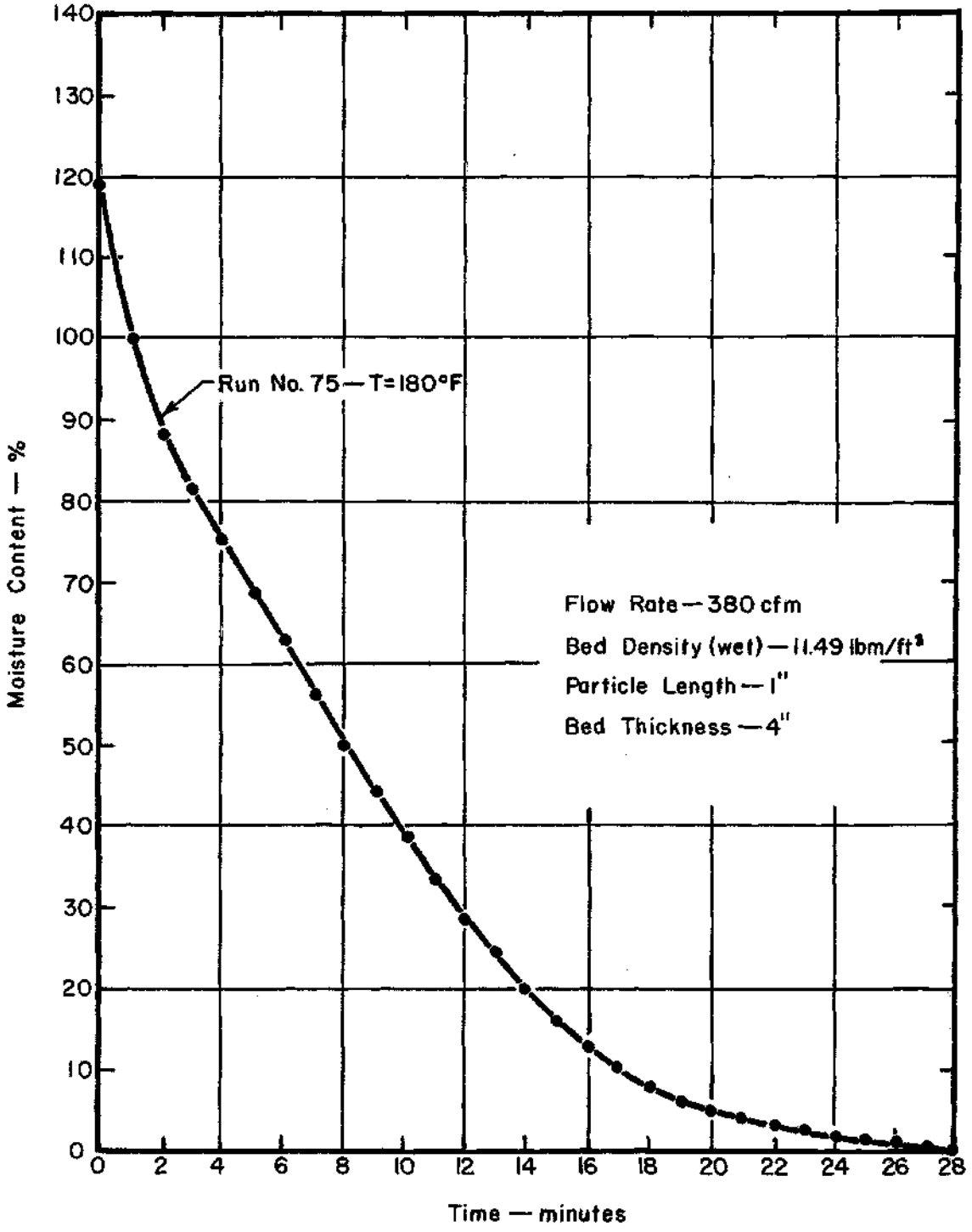


Figure 30b. Drying Curves for Various Temperatures

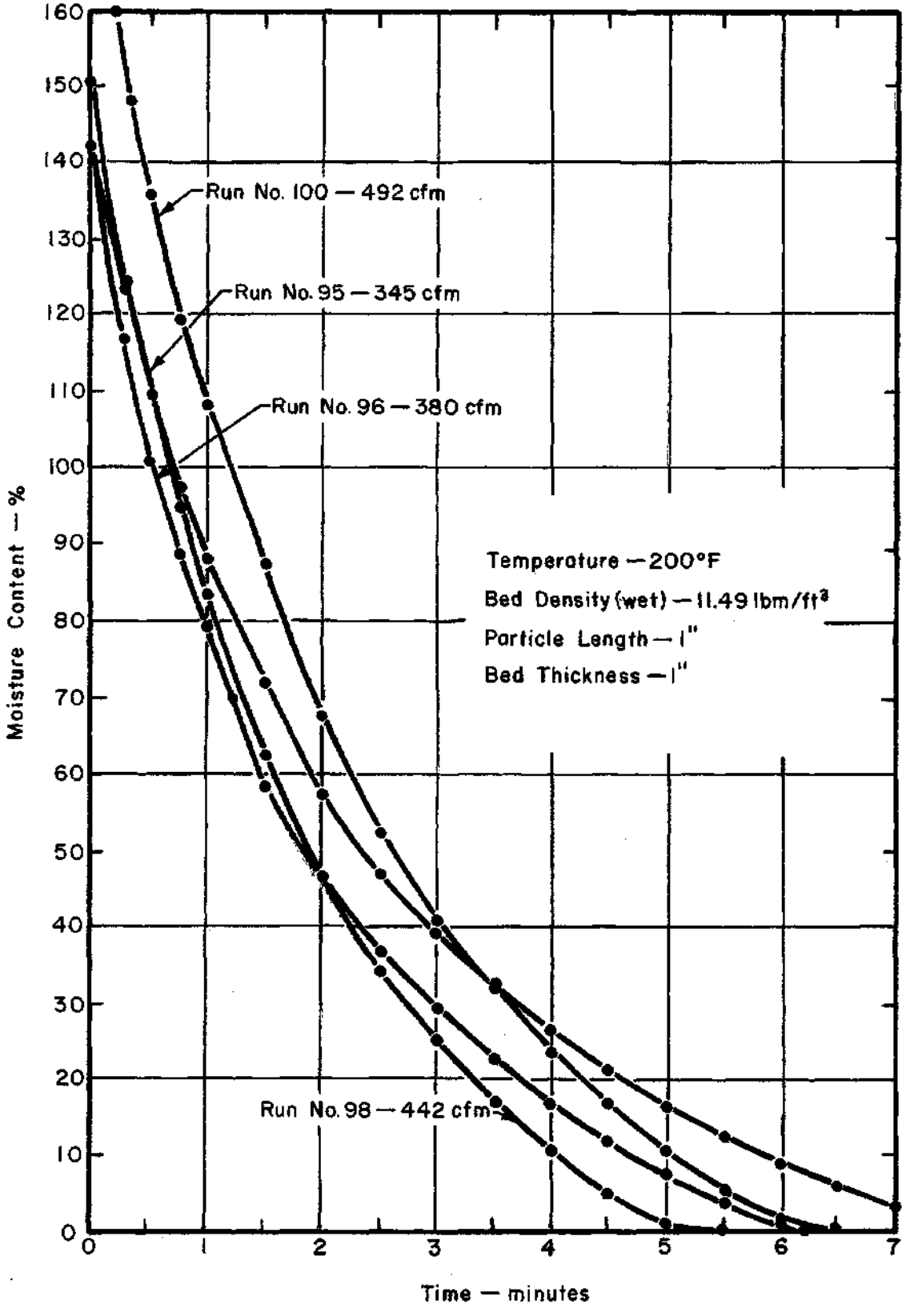


Figure 31. Drying Curves for Various Flow Rates

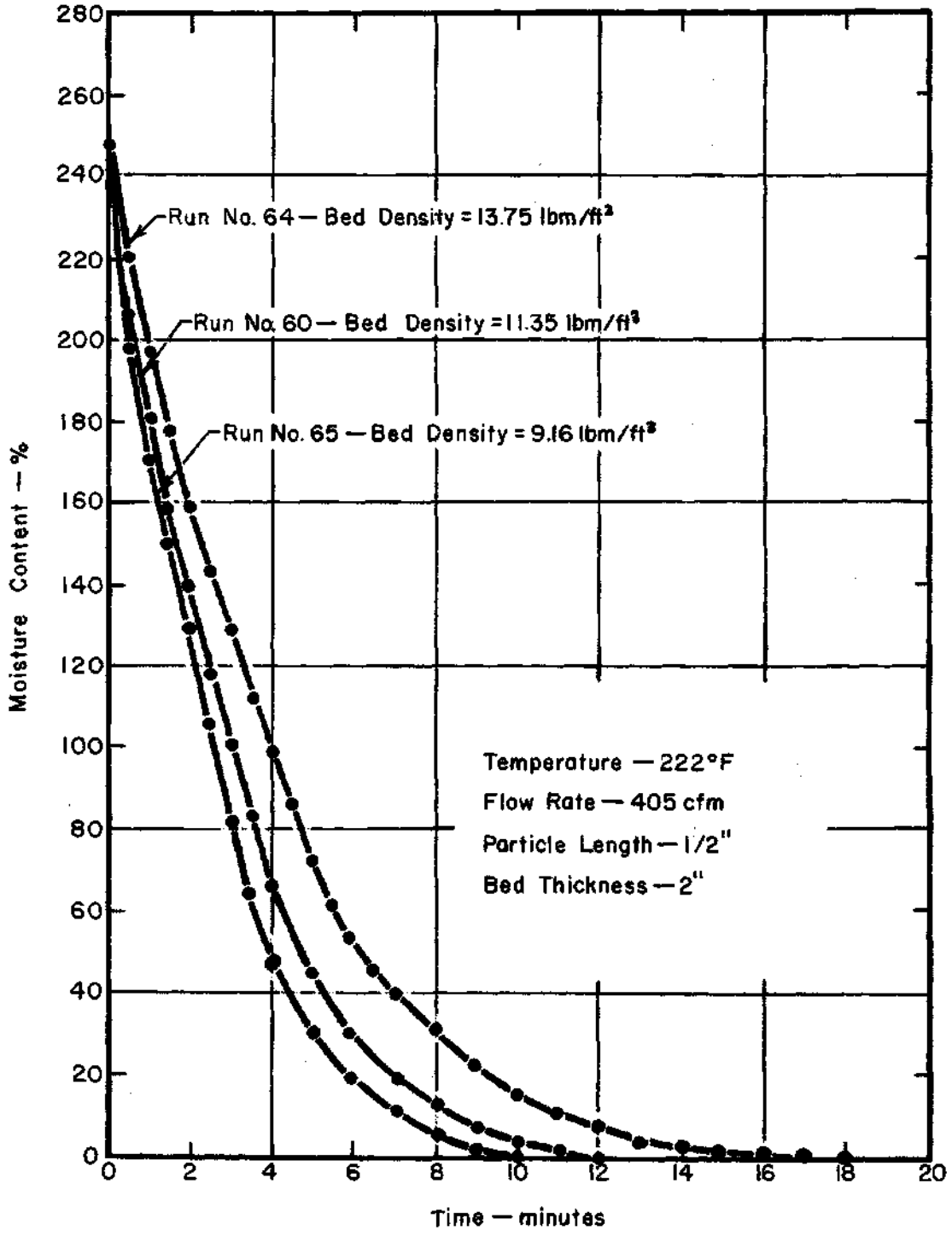


Figure 32. Drying Curves for Various Bed Densities

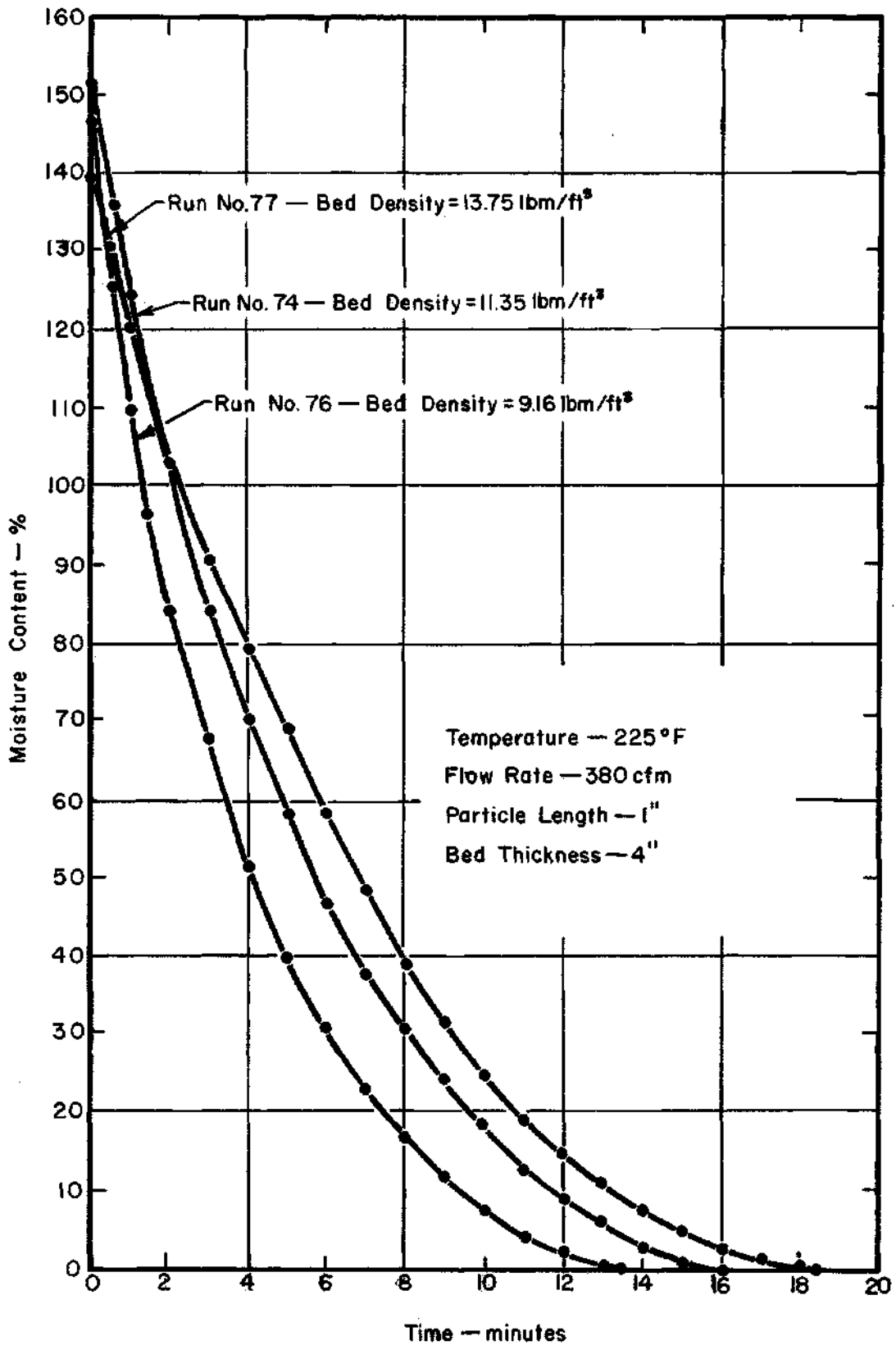


Figure 33. Drying Curves for Various Bed Densities

APPENDIX C

TABULATED DATA

Table 1. Tabulated Test Date

Run Number	Date	Material*	Length at Cutting — Inches**	Length at Drying — Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample*** in H_2O	T_d Room °F	T_w Room °F	T_{duct} °F	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
1	3-18-64	R	16-1	1	29.24	545	1.969	81	60	89	300	4	0.48	392
2	3-18-64	R	16-1	1	29.22	545	1.306	84	61	95	210	3	0.36	392
3	3-18-64	R	16-1	1	29.22	570	1.862	85	61	212	7	1	0.18	392
4	3-24-64	R	16-1	1	29.40	910	1.144	76	64	76	-	1	0.24	M_e
5	3-24-64	R	16-1	1	29.40	845	1.046	76	64	76	-	1	0.24	M_e
6	3-24-64	R	16-1	1	29.40	725	0.837	76	64	76	-	1	0.24	M_e
7	3-24-64	R	16-1	1	29.40	625	0.740	76	64	76	-	1	0.24	M_e

* C.B. — Coastal Bermuda Grass
C — Common Bermuda Grass
R — Rye

** The first term refers to the length of material. The second term refers to the number of cuttings. U refers to uncut material.

*** ΔP refers to the pressure drop across the material only. The pressure across the empty frame was subtracted from the total ΔP.

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room °F	T_w Room °F	T_{duct} °F	Time—Minutes	Bed—Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
8	3-24-64	R	16-1	1	29.40	510	0.434	76	64	76	-	1	0.24	M_e
9	3-24-64	R	16-1	1	29.40	375	0.219	76	64	76	-	1	0.24	M_e
10	3-24-64	R	16-1	1	29.40	755	1.594	74	63	74	-	2	0.48	M_e
11	3-24-64	R	16-1	1	29.40	725	1.480	74	63	74	-	2	0.48	M_e
12	3-24-64	R	16-1	1	29.40	625	1.100	74	63	74	-	2	0.48	M_e
13	3-24-64	R	16-1	1	29.40	510	0.760	74	63	74	-	2	0.48	M_e
14	3-24-64	R	16-1	1	29.40	375	0.484	74	63	74	-	2	0.48	M_e
15	3-24-64	R	16-1	1	29.40	715	1.658	74	63	125	-	2	0.48	M_e
16	3-24-64	R	16-1	1	29.40	690	1.683	74	63	150	-	2	0.48	M_e
17	3-24-64	R	16-1	1	29.40	625	1.326	74	63	150	-	2	0.48	M_e
18	3-24-64	R	16-1	1	29.40	510	0.919	74	63	150	-	2	0.48	M_e

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room ^oF	T_w Room ^oF	T_{duct} ^oF	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
19	3-24-64	R	16-1	1	29.40	375	0.463	74	63	150	-	2	0.48	M_e
20	3-24-64	R	16-1	1	29.40	665	1.696	74	63	200	-	2	0.48	M_e
21	3-24-64	R	16-1	1	29.40	625	1.480	74	63	200	-	2	0.48	M_e
22	3-24-64	R	16-1	1	29.40	510	0.933	74	63	200	-	2	0.48	M_e
23	3-24-64	R	16-1	1	29.40	375	0.494	74	63	200	-	2	0.48	M_e
25	3-24-64	R	16-1	1	29.40	510	0.964	74	63	220	-	2	0.48	M_e
26	3-24-64	R	16-1	1	29.40	375	0.506	74	63	220	-	2	0.48	M_e
27	3-24-64	R	16-1	1	29.40	625	1.451	74	64	220	-	2	0.48	M_e
28	3-24-64	R	16-1	1	29.40	670	1.697	74	64	220	-	2	0.48	M_e
29	3-24-64	R	16-1	1	29.40	720	1.651	74	64	120	-	2	0.48	M_e
30	3-24-64	R	16-1	1	29.40	625	1.223	74	64	120	-	2	0.48	M_e

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room °F	T_w Room °F	T_{duct} °F	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
31	3-24-64	R	16-1	1	29.40	510	0.860	74	64	120	-	2	0.48	M_e
32	3-24-64	R	16-1	1	29.40	375	0.447	74	64	120	-	2	0.48	M_e
34	3-27-64	R	32-1	1	29.26	375	0.671	77	58	200	30	2	0.34	447
35	3-27-64	R	32-1	1	29.28	500	1.588	80	58	200	20	2	0.34	447
36	3-27-64	R	32-1	1	29.28	440	0.908	73	53	200	25	2	0.34	447
37	3-27-64	R	32-1	1	29.25	440	1.328	73	53	200	55	4	0.68	447
38	5-7-64	CB	5-1	1	29.36	300	0.402	78	63	200	15	2	0.25	367
39	5-7-64	CB	5-1	1	29.36	395	0.660	78	64	200	10	2	0.30	367
40	5-7-64	CB	5-1	1	29.36	475	0.780	80	65	200	9	2	0.30	367
41	5-7-64	CB	5-1	1	29.23	375	0.448	81	65	200	12	2	0.30	367
42	5-12-64	CB	7-1	1	29.26	380	1.260	82	71	220	18	4	0.57	321

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room ^oF	T_w Room ^oF	T_{duct} ^oF	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
43	5-12-64	CB	7-1	1	29.29	380	0.900	82	72	220	15	3	0.38	330
44	5-12-64	CB	7-1	1	29.16	380	0.921	78	71	220	10	2	0.30	300
45	5-12-64	CB	7-1	1	29.24	380	0.349	79	70	220	7	1	0.15	300
46	5-14-64	CB	7-1	1	29.15	380	0.592	70	60	220	10	2	0.28	328
47	5-14-64	CB	7-1	1	29.15	380	1.222	71	61	220	20	4	0.67	318
48	5-14-64	CB	7-1	1	29.16	380	0.997	69	58	220	16	4	0.62	318
49	5-19-64	CB	10-1	1	29.45	390	0.781	83	67	156	55	2	0.38	295
50	5-19-64	CB	10-1	1	29.45	390	0.655	87	69	200	15	2	0.38	295
51	5-19-64	CB	10-1	1	29.25	390	0.693	88	70	250	7	2	0.38	295
52	5-19-64	CB	10-1	1	29.35	390	0.764	88	69	306	6	2	0.38	295

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room °F	T_w Room °F	T_{duct} °F	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
53	5-19-64	CB	10-1	1	29.38	390	0.833	90	70	335	3	2	0.38	295
55	5-21-64	CB	11-1	1	29.35	355	0.525	86	69	226	15	2	0.39	285
56	5-21-64	CB	11-1	1	29.35	410	0.600	87	70	230	11	2	0.39	285
57	5-21-64	CB	11-1	1	29.35	440	0.670	89	70	230	9	2	0.39	285
58	5-21-64	CB	11-1	1	29.35	380	0.324	90	70	226	12	2	0.30	300
59	5-21-64	CB	11-1	1	29.34	390	0.556	88	69	224	13	2	0.39	285
60	5-23-64	CB	11-1	0.5	29.38	405	0.818	87	71	225	12	2	0.43	246
61	5-23-64	CB	11-1	1	29.39	405	0.696	88	71	225	13	2	0.43	246
62	5-23-64	CB	11-1	2	29.36	405	0.690	88	70	223	15	2	0.43	246
63	5-23-64	CB	11-1	U**	29.35	405	0.639	87	70	223	16	2	0.43	246

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room ^oF	T_w Room ^oF	T_{duct} ^oF	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
64	5-23-64	CB	11-1	0.5	29.34	405	0.952	84	71	223	15	2	0.52	246
65	5-23-64	CB	11-1	0.5	29.33	405	0.687	84	70	222	10	2	0.35	243
66	5-28-64	C	7-1	1	29.13	405	0.885	92	71	250	10	4	0.83	201
67	5-28-64	C	7-1	1	29.13	405	0.484	91	72	250	7	2	0.42	201
69	6-15-64	CB	14-1	1	29.23	380	1.260	92	75	325	7	3	0.85	163
70	6-15-64	CB	14-1	1	29.23	380	0.783	91	74	325	4	2	0.57	163
71	6-16-64	CB	14-1	1	29.23	380	0.380	93	75	325	3	1	0.29	152
72	6-17-64	CB	14-1	1	29.25	380	1.500	85	72	325	8	4	1.19	152
73	6-17-64	CB	14-1	1	29.25	380	1.455	86	73	275	11	4	1.19	152
74	6-17-64	CB	14-1	1	29.31	380	1.076	87	73	225	15	4	1.19	152
75	6-18-64	CB	14-1	1	29.36	380	0.861	86	75	180	26	4	1.37	119
76	6-18-64	CB	14-1	1	29.26	380	0.683	89	75	225	14	4	0.97	148

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room ^oF	T_w Room ^oF	T_{duct} ^oF	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
77	6-18-64	CB	14-1	1	29.25	380	1.256	92	76	225	18	4	1.50	140
78	6-18-64	CB	14-1	1	29.25	380	0.306	92	76	200	7	1	0.31	142
79	6-18-64	CB	14-1	1	29.25	410	0.380	94	76	200	7	1	0.31	142
80	6-18-64	CB	14-1	1	29.22	440	0.443	94	76	200	7	1	0.31	142
81	6-18-64	CB	14-1	1	29.22	470	0.511	96	76	200	7	1	0.31	142
83	7-3-64	CB	14-1	1	29.10	315	0.361	88	76	88	-	2	0.64	M_e
84	7-3-64	CB	14-1	1	29.10	355	0.482	88	76	88	-	2	0.64	M_e
85	7-3-64	CB	14-1	1	29.10	390	0.485	88	76	88	-	2	0.64	M_e
86	7-3-64	CB	14-1	1	29.10	420	0.559	88	76	88	-	2	0.64	M_e
87	7-3-64	CB	14-1	1	29.10	445	0.643	88	76	88	-	2	0.64	M_e
88	7-3-64	CB	14-1	1	29.10	475	0.723	88	76	88	-	2	0.64	M_e
89	7-3-64	CB	14-1	1	29.10	500	0.820	88	76	88	-	2	0.64	M_e

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room °F	T_w Room °F	T_{duct} °F	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
90	7-3-64	CB	14-1	1	29.10	525	0.902	88	76	88	-	2	0.64	M_e
91	7-3-64	CB	14-1	1	29.10	547	0.989	88	76	88	-	2	0.64	M_e
92	7-3-64	CB	14-1	1	29.10	570	1.078	88	76	88	-	2	0.64	M_e
93	7-3-64	CB	14-1	1	29.10	590	1.162	88	76	88	-	2	0.64	M_e
94	7-3-64	CB	14-1	1	29.10	600	1.211	88	76	88	-	2	0.64	M_e
95	7-6-64	CB	12-2	1	29.28	345	0.247	86	69	200	8	1	0.31	142
96	7-6-64	CB	12-2	1	29.27	380	0.300	86	69	200	7	1	0.31	142
97	7-6-64	CB	12-2	1	29.28	410	0.403	87	70	200	6	1	0.31	142
98	7-6-64	CB	12-2	1	29.28	442	0.483	88	70	200	5	1	0.30	150
99	7-6-64	CB	12-2	1	29.28	470	0.570	89	70	200	6	1	0.33	127
100	7-6-64	CB	12-2	1	29.28	493	0.662	88	69	200	7	1	0.27	178
101	6-18-64	CB	14-1	U	29.25	0	-	-	-	-	2200	-	0.31	142

Table 1. (Continued)

Run Number	Date	Material	Length at Cutting—Inches	Length at Drying—Inches	P (atm) in Hg.	Flow Rate cfm	ΔP Dry Sample in H_2O	T_d Room °F	T_w Room °F	T_{duct} °F	Time—Minutes	Bed Thickness ins	Weight Dry Material lbm	Initial Moisture Content %
102	6-18-64	CB	14-1	U	29.25	0		-	-	-	2200	-	0.31	142
103	7-14-64	CB	14-2	1	29.37	375	0.792	83	70	202	20	4	0.96	213
104	7-14-64	CB	14-2	1	29.36	375	0.407	83	70	202	18	2	0.48	213
105	7-14-64	CB	14-2	1	29.36	375	0.641	85	71	202	19	3	0.72	213
106	7-22-64	CB	15-2	1	29.16	365	0.813	82	75	270	12	4	0.84	257
107	7-22-64	CB	15-2	1	29.16	365	0.602	83	75	270	8	3	0.63	257
108	7-22-64	CB	15-2	1	29.16	365	0.384	84	75	270	4	1	0.21	257
109	7-22-64	CB	15-2	1	29.17	365	0.270	85	76	270	6	2	0.42	257
110	7-22-64	CB	15-2	11	29.17	380	0.817	86	78	325	8	4	0.84	257
111	7-22-64	CB	15-2	1	29.12	V****	1.316	82	73	250	12	4	0.84	257
112	7-22-64	CB	15-2	1	29.13	V	1.274	82	76	250	8	3	0.61	269
113	7-22-64	CB	15-2	1	29.13	V	1.014	83	75	250	4	1	0.21	257
114	7-22-64	CB	15-2	1	29.12	V	1.169	85	75	250	6	2	0.39	284

****V - variable flow rate through bed.

Table 2*. Nutrition Data

Run No.	Material**	Crude Protein %	Crude Fat %	Crude Fiber %
38	C.B.***	22.2	4.5	19.3
40	C. B.	26.9	4.3	19.4
45	C. B.	23.1	3.6	23.3
46	C. B.	22.0	2.9	22.7
49	C. B.	19.7	3.5	20.6
50	C. B.	20.9	3.3	23.9
51	C. B.	19.3	2.9	26.6
53	C. B.	19.1	3.0	25.2
55	C. B.	18.4	3.3	23.6
60	C. B.	17.9	3.1	24.1
61	C. B.	18.6	3.0	24.2
66	C****	17.5	2.5	25.3
67	C	17.3	2.7	24.6
69	C. B.	14.2	3.4	26.9
74	C. B.	17.1	2.9	24.6
101*****	C. B.	8.7	2.1	27.7
106	C. B.	15.4	2.9	28.5
110	C. B.	14.6	2.7	30.2

* Analysis was made by the Georgia Department of Agriculture

** C. B. – Coastal Bermuda Grass
C – Common Bermuda Grass

*** The Coastal Bermuda Grass was fertilized with nitrogen and pot ash

**** The Common Bermuda Grass was not fertilized

***** Sample No. 101 was dried in field.

Table 3. Correlation of Data*

Run No.	Particle Diameter** ft. x 10^3	Re***	f****
38	1.78	50.1	0.571
39	1.78	65.2	0.520
40	1.78	77.3	0.438
41	1.78	61.0	0.550
42	1.79	58.7	0.545
43	1.79	58.7	0.545
44	1.79	58.7	0.560
45	1.79	59.4	0.630
46	1.79	58.7	0.535
47	1.79	58.4	0.550
48	1.79	58.4	0.450
50	1.74	64.0	0.530
51	1.78	56.9	0.616
52	1.78	49.5	0.730
55	1.90	60.3	0.536
56	1.90	66.7	0.495
57	1.90	71.1	0.490
59	1.74	73.0	0.470
60	1.58	56.2	0.570
61	1.78	63.6	0.550
64	1.58	56.2	0.570
69	1.78	47.6	0.788
70	1.78	47.6	0.788
71	1.79	50.5	0.774
72	1.79	49.5	0.769
73	1.79	52.2	0.705
74	1.79	58.8	0.579
75	1.89	73.0	0.481
77	1.89	62.5	0.574
78	1.89	68.6	0.515
79	1.90	74.0	0.555
80	1.90	80.5	0.545
81	1.90	83.5	0.540

*For additional data refer to Table 1.

**Diameter of cylinder having same length and volume as particle.

***See Eq. (26)

****See Eq. (27)

Table 3. (Continued)

Run No.	Particle Diameter ft. x 10^3	Re	f
83	2.07	105.0	0.440
84	2.07	121.5	0.450
85	2.07	134.2	0.438
86	2.07	143.1	0.433
87	2.07	151.2	0.451
88	2.07	162.0	0.447
89	2.07	170.0	0.460
90	2.07	179.0	0.455
91	2.07	186.0	0.465
92	2.07	194.5	0.460
93	2.07	201.0	0.470
94	2.07	205.0	0.470
95	1.82	63.5	0.515
96	1.82	64.3	0.520
97	1.82	69.9	0.594
98	1.82	74.2	0.625
99	1.82	79.7	0.641
100	1.82	82.9	0.680
103	2.21	75.0	0.438
104	2.21	75.0	0.448
105	2.21	74.0	0.466
106	2.21	66.5	0.486
107	2.21	66.8	0.470
109	2.21	55.8	0.498
110	2.21	57.2	0.522
111	2.21	90.3	0.438

Table 4. Comparison of Experimental and Analytical Results for Drying of Coastal Bermuda Grass Between the Initial and Critical Moisture Content

Run No.	C_1* $\left(\frac{1}{\text{ft.min.}}\right)$	C_2* ($\text{ft.}^2 \times 10^6$)	C_3* ($\text{ft.}^2 \times 10^6$)	C_4* $\left(\frac{1}{\text{ft.}}\right)$	C_5*	C_6*	(Experimental) (minutes)	(Analytical)** (minutes)
38	70.5	3.17	12.12	157.0	0.971	0.112	6.41	6.49
39	170.0	3.17	12.12	345.3	0.963	0.135	5.05	5.20
40	206.0	3.17	12.12	353.0	0.965	0.135	3.25	3.51
41	110.0	3.17	12.12	236.0	0.965	0.135	4.11	4.97
42	109.0	3.20	10.70	357.9	0.966	0.112	6.40	6.58
43	137.0	3.20	11.00	305.2	0.970	0.102	4.50	4.12
44	356.0	3.20	10.00	566.0	0.965	0.110	3.60	3.62
45	865.0	3.20	10.00	685.1	0.964	0.110	2.20	1.84
46	211.0	3.20	10.92	330.0	0.967	0.113	3.60	3.60
47	94.8	3.20	10.60	350.0	0.961	0.130	7.50	7.90
48	106.4	3.20	10.60	343.5	0.963	0.121	7.00	6.92
50	230.5	3.03	9.30	490.2	0.955	0.137	3.50	4.73
51	268.5	3.17	9.75	445.0	0.955	0.137	2.45	3.29
52	470.0	3.17	9.75	658.0	0.955	0.137	1.60	2.09
55	285.0	3.61	10.70	560.1	0.954	0.136	4.45	3.85
56	274.0	3.61	10.70	469.3	0.954	0.136	3.40	3.31
57	316.0	3.61	10.70	490.0	0.954	0.110	2.90	3.04
59	317.0	3.03	9.47	584.9	0.954	0.136	3.50	3.86
60	352.0	2.50	6.40	600.0	0.949	0.130	3.60	3.52

* Constants listed on page 84.

** Drying time between the initial and critical moisture content.

Table 4. (Continued)

Run No.	C_1 $(\frac{1}{ft.min.})$	C_2 $(ft.^2 \times 10^6)$	C_3 $(ft.^2 \times 10^6)$	C_4 $(\frac{1}{ft.})$	C_5	C_6	(Experimental) (minutes)	(Analytical) (minutes)
61	329.0	3.17	8.11	562.1	0.949	0.130	4.00	4.00
64	296.1	2.50	6.40	620.0	0.950	0.157	4.30	4.10
69	441.9	3.17	5.39	737.0	0.933	0.113	2.10	2.71
70	698.2	3.17	5.39	785.0	0.933	0.114	1.30	1.71
71	1582.0	3.20	5.06	847.0	0.940	0.108	0.90	1.10
72	304.6	3.20	5.06	662.0	0.955	0.111	3.91	3.66
73	246.3	3.20	5.06	586.0	0.955	0.111	3.00	3.36
74	158.2	3.20	5.06	499.0	0.955	0.111	5.40	4.40
75	111.2	3.57	4.42	440.0	0.919	0.100	6.00	4.93
77	139.0	3.57	5.20	440.0	0.912	0.129	6.10	4.17
78	965.7	3.57	5.27	865.0	0.926	0.108	1.40	1.29
79	1100.0	3.61	5.34	898.0	0.926	0.108	1.39	1.22
80	1310.0	3.61	5.34	959.0	0.919	0.108	1.21	1.10
81	1425.0	3.61	5.34	1018.0	0.917	0.108	1.20	1.09
95	839.1	3.32	4.91	1205.0	0.926	0.108	2.00	2.29
96	989.2	3.32	4.91	845.0	0.926	0.108	1.40	1.44
97	750.3	3.32	4.91	715.0	0.926	0.108	1.41	1.49
98	1390.0	3.32	5.19	1051.0	0.929	0.110	1.40	1.30
99	1156.0	3.32	4.40	775.0	0.922	0.103	0.91	1.05
100	1100.0	3.32	6.15	768.0	0.936	0.118	1.80	1.33
103	86.5	4.89	10.85	348.1	0.943	0.125	7.80	7.40
104	203.0	4.89	10.85	361.0	0.943	0.125	5.50	5.20
105	145.2	4.89	10.85	446.5	0.943	0.125	6.80	6.38
107	141.8	4.89	13.10	318.0	0.951	0.132	2.70	3.20
109	206.5	4.89	13.10	445.0	0.951	0.132	2.20	2.80
110	154.0	4.89	13.10	408.0	0.951	0.132	3.00	3.67

APPENDIX D

COMPUTER PROGRAM

COMPUTER NOMENCLATURE

Computer Notation	Thesis Notation
A	Convenient Group of Variables (see Program)
A1	A
A2	a
B	Convenient Group of Variables (see Program)
B1	B
C	Convenient Group of Variables (see Program)
C1	C
CH	c_h
CPA	$\tilde{C}_{pA}$
CPAL	$\tilde{C}_{pA\ell}$
CPB	$\tilde{C}_{pB}$
D	c
D1	D
DAL	ρ_h
DP	ΔP
DPP	D_p
DW	ρ_w
E	Convenient Group of Variables (see Program)
E1	E
F	Convenient Group of Variables (see Program)
F1	F

Computer Notation	Thesis Notation
G	L
G1	G1
H	distance interval, ft
HFG	H_{fg}
H1	H
K	time interval, minutes
K1	K
KXA	$k_{x,loc}{}^{a}$
L1	L1
M	M
M1	M1
MA	M_A
MH	m_h
MY	M_i
N	number of distance intervals in bed
NB	N_B
NC	G
NN	$\partial M/\partial \tau$
P	P
P1	Initial vapor pressure in free stream, psi
PA	vapor pressure in free stream, psi
PAS	P_{AS}
PASY	Initial vapor pressure at surface of wet material, psi
PR	Pr

Computer Notation	Thesis Notation
PAY	Initial vapor pressure of air in bed, psi
R	R
S	Indexing variable for partial print-out of answers
SC	Sc
SS	f
T	T
TS	$T_s(^oR)$
T1	T_o
TSC	$T_s(^oK)$
TSY	Initial temperature at surface of wet material, ^oR
TY	Initial temperature of air in bed, ^oR
TIMM	τ
V	V
W1	Initial time interval, minutes
W2	Second time interval, minutes
X	τ
X1	Y_1
XA	x_A
XAS	x_{AS}
Y1	Y_2
Y2	Y_3
Z	Y_4

```
BEGIN
COMMENT  INITIAL ADJUSTMENT AND CONSTANT RATE PERIODS IN THROUGH FLOW
         DRYING                                                          ;
         INTEGER L,N; REAL A,B,C,D,E,F,KXA,XAS,XA,CPA,CPB,SC,PR,T,TS,
P,PAS,R,MH,V,DH,DW,M,DAL,DHL,CH,CPAL,MA,NN,K,H,NB,Y1,Y2,X1,A1,A2,Z,B1,
C1,D1,E1,F1,G1,H1,K1,L1,M1,PA,G,PAY,TY,TSY,HFG,NC,MY,TIMM,T1,P1,PASY,
TNEW,TSNEW,MNEW,XANEW,W1,W2,S,SS,DP,DPP,X                                ;
         LIST DATA(N,PAY,TY,TSY,MY,G,NC,W2)                              ;
         FILE IN ABC(1,10)                                               ;
         FORMAT IN COORD(I4,F6.5,3F6.1,F5.3,F6.1,F7.5)                   ;
         READ(ABC,COORD,DATA)          ;
CLOSE(ABC,RELEASE)                                                       ;
BEGIN    ARRAY XA,XAS,T,TS,M,A,B,C,D,E,DAL,DHL,TSC,PAS,F,NN,PA,TNEW,
TSNEW,MNEW,XANEW[0:N]          ;
         LABEL Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9,Q10,Q11,Q12,Q13,Q14,Q15,Q16,
         Q17,Q18,Q19    ;
         FILE OUT JDB 6(2,15)                                            ;
         LIST ANS(L,XA[L],XAS[L],T[L],TS[L],NN[L],M[L]),ANS1(TIMM)       ;
         FORMAT OUT FMT1(X10,"INITIAL ADJUSTMENT AND CONSTANT RATE"),
         FMT2(X10,"PERIODS IN THROUGH FLOW DRYING"),FMT3(X5,"L",X4,"XA",
         X6,"XAS",X6,"T",X7,"TS",X6,"NN",X7,"M"),FMT4(X4,I2,X1,F6.4,X2,
         F6.4,X3,F6.1,X2,F6.1,X2,F6.2,X2,F6.3),FMT5(X22,"TIMM",F10.5)    ;
         WRITE(JDB,FMT1)   ;
         WRITE(JDB,FMT2)    ;
         HFG←1045.0×18.0   ;
         PR←0.720   ;
         CPA←8.118    ;
         CPAL←0.998     ;
         CPB←6.989    ;
         CH←0.250   ;
         DH←65.00    ;
         DW←62.40    ;
         MH←0.840                                                        ;
         V←0.2618    ;
         R←1545.0     ;
         P←14.7×144.0     ;
         MA←18.0    ;
         A1←1.06423320   ;
         B1←1.0137921    ;
         C1←0.0005835                                                    ;
```

```
          D1←4.16385282;
          E1←237.098157          ;
          F1←0.30231574      ;
          G1←0.0033776                                                    ;
          H1←1.152894        ;
          K1←0.745794          ;
          L1←654.2906      ;
          M1←266.778       ;
          T1←543.0    ;
          H←G/N       ;
          P1←0.2000×144.0   ;
          TIMM←0.0          ;
          S←1000.0    ;
          FOR L←1 STEP 1 UNTIL N DO
BEGIN     M[L]←MY/(100.0)          ;
          TS[L]←TSY    ;
          PA[1]←0.200×144.0   ;     T[1]←720.0   ;
          F[1]←1.0-MH/V×(1.0/DH+M[1]/DW)     ;
          W1←0.0001     ;
          K←W1    ;
          D[1]←P/(R×T[1])     ;
          XA[1]←PA[1]/(R×T[1]×D[1])    ;
          NB←NC×P/(R×TY×0.7854)    ;      END;
          FOR L←2 STEP 1 UNTIL N DO
BEGIN     PA[L]←P1    ;      T[L]←T1   ;
          F[L]←1.0-MH/V×(1.0/DH+M[L]/DW)     ;
          D[L]←P/(R×T[L])    ;
          XA[L]←PA[L]/(R×T[L]×D[L])    ;           END;
Q1:       IF ABS(S-1000.0)=0.0
          THEN GO TO Q11
          ELSE GO TO Q12    ;
Q11:
BEGIN     WRITE(JDB,FMT5,ANS1)       ;
          WRITE(JDB[DBL],FMT3)       ;      END                          ;
Q12:      FOR  L←1 STEP 1 UNTIL N DO
BEGIN     T[0]←T[1]   ;
          XA[0]←XA[1]    ;
          F[L]←1.0-MH/V×(1.0/DH+M[L]/DW)                                  ;
          D[L]←P/(R×T[L])    ;
          X←TIMM                                  ;
```

```
          SC←0.680                                                              ;
          DP←1.0-(0.16)×X+0.015×X×(X-1.0)+0.00167×X×(X-1.0)×(X-2.0)-
0.000834×(X-2.0)×(X-1.0)×(X-3.0)×X+0.00025×X×(X-1.0)×(X-2.0)×(X-3.0)×
(X-4.0)       ;
          DPP←(V×(1.0-F[L])×4.0×12.0/(3.14×70000.0))*0.5                        ;
          SS←(1.20×(10.0*6)×P×DP×1.882×DPP×F[L])/(G×(NB*2)×R×T[L]×29.9)         ;
          KXA←SS×NB×0.5×(SC*(-0.667))/(G×5.0)                                   ;
          TSC[L]←1.0/1.8×(TS[L]-492.0)      ;
          Y1←D1×(TSC[L]-187.0)/(TSC[L]+E1)       ;
          X1←0.01×(TSC[L]-187.0)      ;
          Z←-1.87+3.74×(H1-K1×LN(L1/(TSC[L]+M1)+(((L1/(TSC[L]+M1))*2)-1.0
          )*0.5))      ;
          A2←Z*2×(1.87*2-Z*2)/(F1×(1.0+G1×TSC[L]))     ;
          Y2←0.01×3.0×3.0*0.5/(2.0×1.87*3)×(X1-0.01×A2)×(1.87*2-(X1-
          0.01×A1)*2)     ;
          PAS[L]←(10.0*(A1+Y1-B1×(1.0+C1×TSC[L])×Y2))×14.7×144.0      ;
          XAS[L]←PAS[L]/(R×TS[L]×D[L])                                          ;
          E[L]←MH×(CH+M[L]×CPAL)    ;
          A[L]←KXA×(XAS[L]-XA[L])/(1.0-XAS[L])       ;
          C[L]←(XA[L]×(CPA-CPB))+CPB    ;
          B[L]←KXA×C[L]×((SC/PR)*0.667)×(TS[L]-T[L])     ;
          NN[L]←(MA×5.0)/MH×A[L]×V                                              ;
          IF ABS(S-1000.0)=0.0
          THEN GO TO Q7
          ELSE GO TO Q8  ;
Q7:       WRITE(JDB,FMT4,ANS)     ;
Q8:       MNEW[L]←M[L]-K×NN[L]       ;
          TSNEW[L]←TS[L]+((-B[L]-A[L]×HFG)/E[L])×K;
          TNEW[L]←T[L]+(B[L]+CPA×(TS[L]-T[L])×(KXA×(XAS[L]-XA[L])/(1.0-
XA[L])))×K/(D[L]×C[L]×V)-NB×K×(T[L]-T[L-1])/((1.0-XA[L])×D[L]×H)        ;
          XANEW[L]←XA[L]+A[L]×K/(D[L])-NB×K×(XA[L]-XA[L-1])/(D[L]×((1.0
-XA[L])*2)×H)+P×XA[L]×MH×K×NN[L]/(R×T[L]×DW×D[L])+XA[L]×
          (TNEW[L]-T[L])/T[L]    ;      END;
          IF TIMM≥4.0
          THEN GO TO Q4
          ELSE GO TO Q3  ;
Q4:       IF ABS(TSNEW[N]-TS[N])≤0.01
          THEN GO TO Q2
          ELSE GO TO Q3       ;
Q3:       FOR L←1 STEP 1 UNTIL N DO
```

```
BEGIN    M[L]←MNEW[L]
         TS[L]←TSNEW[L]   ;       END
         FOR L←2 STEP 1 UNTIL N DO
BEGIN    T[L]←TNEW[L]
         XA[L]←XANEW[L]   ;              END
         IF ABS(S-1000.0)=0.0
         THEN GO TO Q15
         ELSE GO TO Q16  ;
Q15:     S←0.0  ;
Q16:     K←W1    ;
         TIMM←TIMM+K   ;
         S←S+1.0   ;
         GO TO Q1    ;
Q2:      WRITE(JDB,FMT5,ANS1)      ;
         WRITE(JDB[DBL],FMT3)    ;
         FOR L←1 STEP 1 UNTIL N DO
BEGIN    WRITE(JDB,FMT4,ANS)     ;          END  ;
         END;   END.
```

APPENDIX E

ESTIMATED ERRORS

The errors in this investigation will be classified into three types: (1) Observation errors, made by the observer in reading the instrument. (2) Instrumentation errors, which are made either in the transmission of the signal from the pick-up to the amplifier or from the amplifier to the recorder. (3) Instrument location errors, occurring due to the location of the instrument such as placing a thermocouple in sunlight while measuring air temperature.

The estimate of errors due to the three different types are shown in Table 5. The maximum probable error as defined by Beers[41] is the square root of the sum of the squares of the individual errors.

Table 5. Estimated Probable Errors

Measurement	Observed	Instrumentation	Location	Max. Prob. Error
Flow rate	4%	2%	3%	5.4%
Temperature	2%	1%	0.5%	2.3%
Weighing Section	0	4%	3%	5.0%
Pressure Drop	0.5%	1%	0.5%	1.2%
Dry Weight	0.5%	1%	1%	1.5%
Time	3.0%	0	0	3.0%

Reproducibility of Experimental Results

Due to the nature of the material dried in this investigation it was impossible to run exactly the same test on different days; therefore, all tests to check the reproducibility had to be run using the same material and within a few hours of the first test. In these tests the results were reproduced within five per cent on all samples tested. It is noted that this is not a very accurate check on the reproducibility of the results; however, due to the variation in initial moisture content and material size, this was the best procedure available.

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VITA

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